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STABILITY AND TRANSITION OF THREE-DIMENSIONAL BOUNDARY LAYERS

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Abstract The recent progress in three-dimensional boundary-layer stability and transition is reviewed. The material focuses on the crossflow instability that leads to transition on swept wings and rotating disks. Following a brief overview of instability mechanisms and the crossflow problem, a summary of the important findings of the 1990s is given.

1. INTRODUCTION

In low-disturbance environments such as flight, boundary-layer transition to turbulence generally occurs through the uninterrupted growth of linear instabilities. The initial conditions for these instabilities are introduced through the receptivity process, which depends on a variety of factors (Saric et al. 2002). There is no shortage of publications in the field of boundary-layer stability and transition. Comprehensive reviews for both two- and three-dimensional (3-D) flows are given by Arnal (1994), Mack (1984), and Reshotko (1994). Reed et al. (1996) give an up-to-date discussion of the effectiveness and limitations of linear theory in describing boundary-layer instabilities. The reader is referred to these reports for overviews of much of the early work in stability and transition. Crouch and coworkers (Crouch 1997, Crouch & Ng 2000, Crouch et al. 2001) and Herbert (1997b) have presented analyses of transition with specific applications to flight.

Reed & Saric (1989) review the early theoretical and experimental work in 3-D boundary layers. Swept wings, rotating disks, axisymmetric bodies (rotating cones and spheres), corner flows, attachment-line instabilities, as well as the stability of flows for other 3-D geometries are covered. This historical account and literature survey of the crossflow instability are used for references prior to 1989. During the
past 12 years, most of the progress has occurred in rotating-disk and swept-wing flows, and this review concentrates on these areas.

The study of 3-D boundary layers is motivated by the need to understand the fundamental instability mechanisms that cause transition in swept-wing flows. Research has identified four types of instabilities for these flows: attachment line, streamwise, centrifugal, and crossflow. The attachment-line boundary layer can undergo an instability or be subject to contamination by wing-root turbulence; these phenomena are associated with, in general, swept wings with a large leading-edge radius (Hall et al. 1984, Reed & Saric 1989). The streamwise instability gives the familiar Tollmien-Schlichting (T-S) wave in two-dimensional flows (Reshotko 1976, Mack 1984, Reed et al. 1996). Centrifugal instabilities appear over concave regions on the surface and result in the development of Görtler vortices (Saric 1994). The crossflow instability occurs in regions of pressure gradient on swept surfaces or on rotating disks. In the inviscid region outside the boundary layer, the combined influences of sweep and pressure gradient produce curved streamlines at the boundary-layer edge. Inside the boundary layer, the streamwise velocity is reduced, but the pressure gradient is unchanged. Thus, the balance between centripetal acceleration and pressure gradient does not exist. This imbalance results in a secondary flow in the boundary layer, called crossflow, that is perpendicular to the direction of the inviscid streamline. The 3-D profile and resolved streamwise and crossflow boundary-layer profiles are shown in Figure 1.

Because the crossflow velocity must vanish at the wall and at the edge of the boundary layer, an inflection point exists and provides a source of an inviscid instability. The instability appears as co-rotating vortices whose axes are aligned to within a few degrees of the local inviscid streamlines.

Unlike T-S instabilities, the crossflow problem exhibits amplified disturbances that are stationary as well as traveling. Even though both types of waves are present in typical swept-wing or rotating-disk flows, transition is usually caused by either one, but not both, of these waves. Although linear theory predicts that the traveling disturbances have higher growth rates, transition in many experiments is induced by stationary waves. Whether stationary or traveling waves dominate is related to the receptivity process. Stationary waves are more important in low-turbulence environments characteristic of flight, whereas traveling waves dominate in high-turbulence environments (Bippes 1997, Deyhle & Bippes 1996).

Stationary crossflow waves (\(v'\) and \(w'\) disturbances) are typically very weak; hence, analytical models have been based on linear theory. However, experiments often show evidence of strong nonlinear effects (Dagenhart et al. 1989, 1990; Bippes & Nitschke-Kowsky 1990; Bippes et al. 1991; Deyhle et al. 1993; Reibert et al. 1996). Because the wave fronts are fixed with respect to the model and are nearly aligned with the potential-flow direction (i.e., the wavenumber vector is nearly perpendicular to the local inviscid streamline), the weak \((v', w')\) motion of the wave convects \(O(1)\) streamwise momentum producing a strong \(u'\) distortion
in the streamwise boundary-layer profile. This integrated effect and the resulting local distortion of the mean boundary layer lead to the modification of the basic state and the early development of nonlinear effects.

An interesting feature of the stationary crossflow waves is the creation of secondary instabilities. The \( u' \) distortions created by the stationary wave are time independent, resulting in a spanwise modulation of the mean streamwise velocity profile. As the distortions grow, the boundary layer develops an alternating pattern of accelerated, decelerated, and doubly inflected profiles. The inflected profiles are inviscidly unstable and, as such, are subject to a high-frequency secondary instability (Kohama et al. 1991, Malik et al. 1994, Wassermann & Kloker 2002).

Figure 1  Swept-wing boundary-layer profiles.
This secondary instability is highly amplified and leads to rapid local breakdown. Because transition develops locally, the transition front is nonuniform in span and characterized by a “saw-tooth” pattern of turbulent wedges.

Arnal (1997), Bippes (1999), and Kachanov (1996) review the European contributions to stability and transition in 3-D boundary layers and, as such, are companion papers to this work. Recently, improvements in both experimental techniques and computational methods have opened the door to a new understanding of transition in 3-D boundary layers. This review focuses on the latest developments, with emphasis on the experimental work and relevant comparisons with computational fluid dynamics (CFD).

2. ROTATING DISK

The boundary layer on the surface of a rotating disk has been used as a model problem for swept-wing flow because the velocity profiles on the disk provide a basic state and stability behavior very similar to that of the swept-wing boundary layer shown in Figure 1. The advantage of the rotating disk is the existence of a well-known similarity solution for the basic flow that features a boundary layer of constant thickness. Reed & Saric (1989) reviewed in detail the progress on the rotating-disk problem through the 1980s. At the time of that review, a number of analyses and experiments were beginning to provide a good picture of the linear stability of stationary waves, especially those connected to the upper inviscid branch of the neutral stability curve, but much remained to be determined about the lower viscous branch, traveling waves, and nonlinear behavior.

Linear stability of traveling waves was addressed by Balakumar & Malik (1991), who used a parallel-flow approximation and considered the neutral curves at nonzero frequencies. They found that, as the frequency increases, the Reynolds number of the lower minimum decreases while that of the upper minimum increases. The minimum critical Reynolds number over all frequencies is found to be a lower-branch minimum. For the frequencies examined, the Reynolds number and wave angle associated with the upper-branch minimum do not change significantly, but there are large changes at the lower-branch minimum.

For the linear problem, Faller (1991) considered both the upper and lower branches of the rotating-disk neutral curve and found minimum critical Reynolds-number behavior equivalent to that found by Malik (1986) for the stationary upper-branch mode and similar to the results obtained by Balakumar & Malik (1991) for the traveling lower-branch mode. Faller (1991) explained that the lower-branch modes are typically not observed in experiments both because they have a lower growth rate than those on the upper branch and because, among the lower-branch modes, stationary modes are much less amplified than traveling modes. This is in contrast to the upper branch where the amplification of stationary waves is nearly equal to the most amplified traveling waves. Because most experiments are better able to detect stationary waves than traveling waves, it is natural that slowly
growing traveling disturbances like the lower-branch modes can easily escape detection.

Jarre et al. (1996a) measured traveling and stationary waves on a rotating disk to confirm linear-stability theory (LST) predictions, particularly those concerning traveling waves. The fluctuation intensity measured close to the upper-branch stationary-wave neutral point is 2%, so the experiments have a relatively high disturbance level. Jarre et al. (1996a) found a minimum critical Reynolds number for stationary waves reasonably close to the theoretical predictions. The agreement between the measurements and theory for the corresponding wavenumber and the disturbance-mode eigenfunctions (Malik 1986) is not as good. This appears to be due to the basic-state circumferential velocity not going to zero above the disk as the similarity solution requires.

A somewhat different approach to the rotating-disk problem is taken in a series of papers by Itoh (1994, 1996, 1997, 1998), Buck & Takagi (1997), Takagi et al. (1998, 2000), and Buck et al. (2000). Itoh (1994) formulated the linear-stability problem to explicitly include the curvature of the inviscid streamline at the boundary-layer edge. He found that strongly curved inviscid streamlines produce a centrifugal-type instability similar to the Görtler instability on concave walls (Saric 1994). Because streamlines are highly curved near the symmetry axis in rotating disk flow, Itoh (1996, 1998) investigated whether the streamline-curvature instability can occur in these boundary layers. He found that the linear-stability neutral curve has two lobes: one for the crossflow instability and the other for the streamline-curvature instability. The streamline-curvature instability is unstable at much lower Reynolds numbers than crossflow, and the range of unstable wave numbers decreases as the Reynolds number grows. For crossflow, the unstable wave-number range increases with increasing Reynolds number past the minimum critical value. Itoh (1996, 1998) concluded that the streamline-curvature instability is the source of the lower branch of the neutral curve discussed above.

Although the streamline-curvature instability is destabilized at a much lower Reynolds number than the crossflow instability, in most experiments the crossflow instability is observed. This discrepancy is likely a result of either smaller growth rates or smaller receptivity of the streamline-curvature instability versus the crossflow instability. To address the growth-rate question, Itoh (1997) calculated that, in a region unstable to both instabilities, the growth of streamline-curvature modes is larger than that of crossflow modes. In a subsequent experiment, Takagi et al. (1998) confirmed, at least qualitatively, the predictions by Itoh (1997). Experimental verification is also provided by Buck & Takagi (1997) and Takagi et al. (2000), who used point-source disturbance generators. Buck et al. (2000) also demonstrated that for point-source forcing early growth of streamline-curvature disturbances can suppress the appearance of crossflow disturbances that are observed under zero-forcing conditions. However, Buck et al. (2000) suggested that the suppression may not prevent or delay transition because the large-amplitude streamline-curvature disturbances may lead to turbulence.
2.1. Nonlinearities, Wave Interactions, and Secondary Instabilities

When the review by Reed & Saric (1989) was published there was only a limited body of work that considered the nonlinear behavior of finite-amplitude disturbances in rotating-disk boundary layers. Subsequently, there have been several analytical and experimental investigations of the nonlinear regime. One of the analytical works was by Vonderwell & Riahi (1993), who examined the weakly nonlinear behavior of waves near the minimum critical Reynolds number of the upper-branch neutral point. They developed a Landau-type equation for the wave amplitude and found that the Landau constant is negative such that there is a subcritical instability at \( R < R_c \) and that disturbance amplitudes go to infinity (in the weakly nonlinear model) in finite time for \( R > R_c \). This conflicts with the earlier finding by Itoh (1985) stating that the instability is supercritical. Jarre et al. (1996b) conducted a follow-up experiment to their previous work (Jarre et al. 1996a) that determined the Landau constant from data. The experiment features a single, large-amplitude roughness element that generates finite-amplitude stationary waves. Even though the basic state differs from that of the similarity solution as mentioned above, the results do seem to support at least a qualitative analysis of the Landau equation coefficients. In contrast to the results presented by Vonderwell & Riahi (1993) that indicated a negative Landau constant, Jarre et al. (1996b) found that the Landau constant is positive, meaning that the instability is supercritical. This result seems reasonable in light of the related swept-wing experiments discussed below that show that finite-amplitude crossflow waves undergo amplitude saturation.

Corke & Knasiak (1998) considered nonlinear wave interactions on a rotating disk. They used a polished disk with various periodic roughness configurations located near the minimum critical Reynolds number, \( R_c \), for upper-branch modes. Various tests showed that stationary modes obeyed linear theory in terms of circumferential wave number, growth rate, and mode shape up to a radius of \( R/R_c \approx 1.5 \). Traveling waves are also well predicted by linear theory. In the nonlinear regime, Corke & Knasiak (1998) found significant growth of very-low-wave-number stationary waves that are obviously not growing according to linear theory. Using the bicoherence of traveling and stationary wave phases, they confirmed a triad resonance. In all of the roughness configurations tested (including no artificial roughness), low circumferential wave numbers were produced by triad resonance and grew to at least as large an amplitude as the most amplified stationary disturbances.

A mechanism for the final stage of the transition process, breakdown of the laminar boundary layer by means of a secondary instability, was considered by Balachandar et al. (1992) using a Floquet approach that considered a steady basic state in the rotating reference frame consisting of the mean flow plus the most amplified stationary disturbance predicted by linear theory at amplitudes ranging from 6% to 12%. Below 9% amplitude, no secondary modes are destabilized, but above this threshold, a broad band of secondary modes are destabilized with growth rates as high as four times that of the stationary waves of the primary instability.
2.2. Absolute Instabilities

Perhaps the most important recent development in the study of the rotating-disk problem is the theory by Lingwood (1995) that rotating-disk boundary layers support an absolute instability at $R = 510$. The implication of this finding is that the steady, laminar basic state cannot exist beyond this Reynolds number regardless of the care taken in preparing and conducting the experiment. This finding is in good agreement with various rotating-disk experiments, most of which do not maintain laminar flow much beyond $R = 500$.

Lingwood (1995) followed Briggs’ method (see Huerre & Monkewitz 1990), which prescribes the means to evaluate the Fourier-Laplace integral that arises from considering an initial-boundary-value problem for impulsive forcing in a developing flow. If the group velocity of a disturbance wave packet goes to zero while the temporal growth rate is positive, then the flow is absolutely unstable. Lingwood (1995) applied Briggs’ method by using a parallel-flow approximation and observed a pinch between two neutral branches of the dispersion relationship that yields a positive temporal growth rate at a Reynolds number $R \geq 510.625$. As $R$ is increased, the absolutely unstable wave-number range expands and at large $R$ tends toward the limits found for the inviscid case. Lingwood (1995) concluded that the absolute instability arises from an inviscid mechanism and that the good correlation between the Reynolds-number limit for absolute instability and the experimental transition Reynolds numbers makes a case for transition being triggered by the absolute instability.

Lingwood (1996) then performed an experiment on a rotating disk in which stationary-wave development leading to transition was observed with both an unexcited and an excited flow in which a once-per-revolution transient disturbance was introduced near the center of the disk. The unexcited flow exhibited stationary waves that grew in a manner consistent with LST and led to transitional behavior by $R = 514$ with fully turbulent behavior observed by approximately $R = 600$. Perhaps the most important information from the tests without excitation is that the spectra of the time series do not show any evidence of secondary instabilities prior to breakdown. Balachandar et al. (1992) claimed that an amplitude of at least 9% would be required to see such behavior and that Lingwood’s (1996) stationary waves only reach 3% prior to transition, so this result is not unexpected. The other interesting feature of the spectra is that there is an abrupt jump of approximately a factor of 10 in the fluctuation amplitudes at all frequencies when $R$ is between 497 and 514 that is not preceded by any significant signs of imminent breakdown to turbulence. In fact, downstream of the jump, the spectra do not appear turbulent; there is a distinct spectral peak associated with the stationary waves and a lower amplitude peak associated with harmonics of the stationary waves. The jump occurs exactly in the region of the absolute instability identified by Lingwood (1995).

A more compelling demonstration of the existence of the absolute instability comes from Lingwood’s (1996) experiments that include transient forcing. The leading edge of the wave packet convects in the radial direction at an essentially
constant rate, whereas the propagation rate of the trailing edge decreases dramatically as the packet approaches the critical Reynolds number for the absolute instability, as predicted by Lingwood’s earlier analysis. Taken together, the experimental evidence convincingly shows that for cases with low-amplitude initial disturbances (i.e., low enough to avoid secondary instabilities) disturbances pass from a convective-instability regime for \( R < 510 \) to an absolutely unstable regime for \( R > 510 \). The discovery that an absolute instability exists on rotating disks is important because this places a firm upper bound on the Reynolds numbers for which laminar flow is possible.

The improved picture of rotating-disk instabilities includes a number of routes to turbulence. First, high-amplitude traveling disturbances (if they exist) may be amplified at very low Reynolds numbers. At higher Reynolds numbers, stationary waves dominate. These can lead to turbulence through two distinct mechanisms. If the disturbance levels are high, then high-frequency secondary instabilities lead to rapid transition. If disturbance levels are low and secondary instabilities do not appear, the flow becomes absolutely unstable for Reynolds numbers above 510, and transition occurs. Despite a better understanding of this basic scenario, much remains to be learned, especially in regard to nonlinear behavior. One important problem that remains is to conclusively identify whether the various branches of the instability are subcritical or supercritical. Another unresolved question is what effect, if any, finite-amplitude disturbances have on the absolute instability.

3. SWEPT WING

3.1. Swept Flat Plate

A prototype of the swept wing is a swept flat plate with a pressure body (Saric & Yeates 1985) that could be used to verify the basic aspects of LST. The advantage of the flat plate is that the first neutral point can be placed away from the leading edge and thus avoiding nonparallel and curvature effects. In a series of experiments on swept flat plates (Nitschke-Kowsky & Bippes 1988, Müller 1990, Deyhle et al. 1993, Kachanov & Tararykin 1990, Kachanov 1996), the observed wavelength and growth rate of the crossflow wave is in general agreement with linear theory (Orr-Sommerfeld solutions).

Deyhle et al. (1993) and Kachanov (1996) developed techniques to create controlled traveling waves within the boundary layer and observed the growth of them. Linear theory was also verified in this case. Bippes (1997) put into perspective the importance of traveling modes and their dependence on freestream conditions. In general, under conditions of high freestream turbulence, traveling modes dominate and linear theory predicts the behavior of the crossflow waves. In low-turbulence flows, stationary waves dominate, the mean flow is distorted, and nonlinear behavior occurs rapidly, thus obviating the linear analysis beyond initial mode selection.
3.2. Receptivity

When the paper by Reed & Saric (1989) was published, very little was known regarding receptivity of 3-D boundary layers. It was acknowledged that receptivity must play a major role in determining the details of crossflow transition because of the wide range of behaviors observed using different models in various experimental facilities. In detailing a list of unanswered questions regarding these variations observed in different facilities, Reed & Saric (1989) concluded, “All of this serves notice that stability and transition phenomena are extremely dependent on initial conditions.” Since then, a significant research effort that includes experimental, theoretical, and computational work has made great progress toward an understanding of the applicable receptivity processes [the reader is referred to the review by Bippes (1999) for additional details]. Owing to space constraints, we focus primarily on the recent efforts in this area.

ROLE OF FREESTream FLUCTUATIONS  The effect of freestream turbulence on crossflow transition was investigated by Deyhle & Bippes (1996), who performed transition measurements on a crossflow-dominated swept-plate model in a number of different wind-tunnel facilities with varying freestream-turbulence levels. They found that for the particular model used in those experiments turbulence intensities above $Tu = 0.0015$ produced transition behavior dominated by traveling waves, but that for lower turbulence levels stationary waves dominated. It is surprising to note that for increased turbulence levels where traveling waves dominate but the turbulence intensity is not too high, $0.0015 < Tu < 0.0020$, transition was actually delayed relative to low-turbulence cases at the same Reynolds number. The explanation for this is that the traveling waves excited by the increased freestream turbulence were sufficiently strong to prevent stationary waves from causing transition but were not strong enough to cause transition as quickly as the stationary waves they replaced. This behavior indicates that transition results from many wind tunnels may have no bearing on flight results because quite low levels of turbulence are sufficient to generate traveling-wave-dominated behavior, counter to the stationary-wave-dominated behavior observed in flight.

In another experiment, Radeztsky et al. (1999) found that transition behavior on a swept wing is insensitive to sound, even at amplitudes greater than 100 dB. The conclusion is that the variations observed by Deyhle & Bippes (1996) at varying levels of $Tu$ are due primarily to variations in the vortical components of the freestream fluctuations and not to the acoustic component.

ROLE OF SURFACE ROUGHNESS  The receptivity mechanism for the stationary vortices that are important for transition in environments with very-low-amplitude turbulent fluctuations (i.e., characteristic of the flight environment) is surface roughness. This was conclusively established by Müller & Bippes (1989), who translated a swept-flat-plate model relative to a wind-tunnel test section and found that the recurring stationary transition pattern was fixed to the model. The instability features they observed had to be related to model roughness rather than
to fixed features of the freestream flow generated by nonuniformities of the screens or other effects.

Detailed roughness studies of isolated 3-D roughness features have been completed by Juillen & Arnal (1990), who found that for isolated roughness elements the correlation by von Doenhoff & Braslow (1961) describing the limit for bypass transition is correct. Radeztsky et al. (1999) showed that the characteristics of isolated 3-D roughness elements whose $Re_k$ values fall below the bypass-inducing level play a very important role in transition behavior. Radeztsky et al. (1999) found that roughness is most effective at generating crossflow disturbances at or just upstream of the first neutral point, that the transition location is quite sensitive to roughness height even for roughness Reynolds numbers as low as $Re_k = 0.1$, and that the roughness diameter must be greater than 10% of the most amplified stationary wavelength to be effective.

In addition to isolated 3-D roughness, natural surface roughness can also play a significant role in transition location. Radeztsky et al. (1999) found that a decrease in surface-roughness amplitude from 9.0 $\mu$m rms to 0.25 $\mu$m rms increases the transition Reynolds number by 70%. In contrast to this, experiments by Reibert et al. (1996) showed that an artificial distributed roughness array with an amplitude of 6 $\mu$m or greater applied near the leading edge produces transition behavior almost completely insensitive to roughness amplitude in the range of 6–50 $\mu$m. In the experiment by Reibert et al. (1996), the periodic roughness arrays concentrate the disturbances in a very narrow band of wavelengths, and these well-defined modes generally lead to nonlinear amplitude saturation of the most amplified wave. Amplitude saturation renders the initial amplitude nearly unimportant so long as it is above a threshold for which saturation occurs prior to transition. In other words, Radeztsky et al. (1999) saw a strong roughness-amplitude effect for roughness conditions that did not lead to saturation, whereas Reibert et al. (1996) did not see a strong roughness-amplitude effect for roughness at or above saturation-producing amplitudes.

**RECEPTIVITY COMPUTATIONS** A number of theoretical and computational approaches to swept-wing crossflow receptivity have been applied. Some of the more recent include an adjoint equation approach by Fedorov (1989), a parabolized stability equation (PSE; linear PSE, LPSE; nonlinear PSE, NPSE) approach by Herbert & Lin (1993), and a direct numerical simulation (DNS) approach by Spalart (1993). Other efforts are by Crouch (1993, 1994, 1995) and Choudhari (1994), both consider the receptivity of Falkner-Skan-Cooke boundary layers as perturbations of a parallel boundary layer. The framework of their approaches allows both surface roughness and freestream acoustic disturbances to be considered as disturbance sources. Choudhari (1994) extended his work to include acoustic-wave-angle effects and a variety of different roughness configurations including isolated roughness, roughness arrays and lattices, as well as distributed random roughness. Crouch (1994) emphasized a framework equally applicable to T-S and crossflow disturbances. Because traveling-wave receptivity scales with two small
parameters (the freestream velocity-fluctuation amplitude and surface-roughness amplitude), whereas the stationary-wave receptivity scales with only one (the surface roughness), both authors note that stationary waves should be expected to dominate in low-disturbance environments and that traveling waves should only appear for large freestream fluctuation levels. The experiments by Radeztsky et al. (1999) confirm that acoustic forcing is not an effective receptivity source when the surface roughness is low.

The method described by Crouch (1994) is used by Ng & Crouch (1999) to model the artificial roughness arrays used in the Arizona State University (ASU) swept-wing experiments by Reibert et al. (1996). Ng & Crouch (1999) give results that are in good agreement with the experiments when the receptivity of 12-mm waves (most unstable mode) to low-amplitude (roughness height 6 or 18 µm), 12-mm-spaced roughness arrays is considered. However, the linear theory over-predicts the receptivity of the 48-µm roughness, suggesting the nonlinear effects reduce receptivity effectiveness. Results are not as good for the receptivity of 9-, 12-, and 18-mm waves to 36-mm-spaced roughness, but this problem may be complicated by the nonlinear growth and production of harmonics associated with the stationary crossflow waves. Bertolotti (2000) presented another Fourier-transform approach that includes the effects of basic flow nonparallelism. Bertolotti applied the method to both swept Hiemenz flow and the Deutschen Centrum für Luft- und Raumfahrt (DLR) swept-flat-plate experiment by Bippes and coworkers. Comparisons between the receptivity predictions and the DLR swept-plate results are quite good.

Another recent approach by Collis & Lele (1999) consists of solving the steady Navier-Stokes equations in the leading-edge region of a swept parabolic body and of using that solution as a basic state for a linearized steady-disturbance system that includes surface roughness. Comparing the results of this approach to those obtained by Choudhari (1994) and Crouch (1994) shows that receptivity to surface roughness is enhanced by convex surface curvature and suppressed by non-parallelism. Neglecting nonparallelism causes the local approach to overpredict receptivity by as much as 77% for the most amplified stationary crossflow wave. The error introduced by neglecting nonparallelism is most severe for wavelengths in the range most amplified by the crossflow instability and for roughness close to the first neutral point. Janke (2001) confirmed overprediction of receptivity by local approaches. The implication is that amplitude-based transition-prediction methods need to employ a receptivity model that includes nonparallelism because the crossflow modes that dominate transition are most strongly affected by this influence. This result stands somewhat in contradiction to the result by Ng & Crouch (1999) that includes neither surface curvature nor nonparallelism yet shows good agreement with experimental results.

ROLE OF TURBULENCE/ROUGHNESS INTERACTIONS Although receptivity models have considered the interaction of surface roughness with acoustic fluctuations, none consider the receptivity of freestream turbulence interacting with surface
roughness. The experiments by Radeztsky et al. (1999) and Deyhle & Bippes (1996) would suggest however that the turbulent fluctuations play a much more significant role in the transition process than acoustic fluctuations. Deyhle & Bippes (1996) give a turbulence intensity criterion that selects traveling or stationary modes, but drawing an analogy to the acoustic/roughness interaction results, one should suspect that it is the interaction of freestream turbulence with surface roughness that is the important consideration. Turbulence level alone may not be sufficient to predict whether traveling or stationary waves will be most important.

With this in mind, the experiment by White et al. (2001) examined the interactions of surface roughness and freestream turbulence and confirmed that the selection of traveling-wave or stationary-wave behavior is not as straightforward as the $Tu = 0.0015$ criterion. In that experiment, a swept-wing model outfitted with a variable-amplitude roughness insert at $x/c = 0.025$ (near the first crossflow neutral point) was used to change the roughness configuration during an experiment from nominally smooth (<0.5 $\mu$m rms) to an 8-mm-spaced, 50-$\mu$m-high roughness pattern with 2-mm diameter roughness elements. A turbulence-generating grid provided a turbulence intensity of $Tu = 0.003$. With a nominally smooth leading edge, a sharp saw-tooth transition pattern was observed in a naphthalene flow-visualization experiment, indicating stationary-wave-dominated transition, but when the roughness array was activated, the saw-tooth transition front was replaced by a diffuse spanwise-invariant transition front indicative of traveling-wave-dominated transition. The implication is that traveling waves do result from an interaction of freestream velocity fluctuations with surface roughness and not from turbulence intensity alone. Therefore, even though stationary waves are generated by surface roughness, increasing the roughness amplitude does not necessarily make stationary waves more likely to be observed than traveling waves. Instead, both surface roughness and turbulence intensity must be considered together in a more sophisticated manner.

3.3. Nonlinear Behavior

NONLINEAR SATURATION  In all the experiments by Bippes and coworkers, the growth of the stationary and traveling crossflow waves showed initial qualitative agreement with linear theory. However, the disturbance amplitude saturated owing to nonlinear effects. Also, the amplitude of the traveling waves showed a spanwise modulation indicating nonlinear interactions with the stationary modes. The streamwise-mode shapes had saturation amplitudes of approximately 20%.

Reibert et al. (1996) investigated the nonlinear saturation of stationary waves using micron-sized artificial roughness elements to control the initial conditions. Full-span arrays of roughness elements were used to preserve the uniform spanwise periodicity of the disturbance as shown in the velocity contour plot of Figure 2, where the crossflow moves from left to right.

The development of crossflow occurs in two stages. The first stage is linear and is characterized by small vertical $v'$ and spanwise $w'$ disturbance velocities
convecting low-momentum fluid away from the wall and high-momentum fluid toward the wall. This exchange of momentum occurs in a region very close to the wall where there are large vertical gradients in the basic-state streamwise velocity. Because of this large gradient, the small displacements caused by the $v'$ and $w'$ disturbance components quickly lead to large disturbances $u'$ superposed on the basic state further downstream. This $u'$ component soon becomes too large, and nonlinear interactions must be included in any calculations. This is the second stage, evidenced by rollover seen in the streamwise-velocity contours. By forcing the most unstable mode (according to linear theory), nonlinear saturation of the disturbance amplitude is observed well before transition. Although the initial growth rate increases with increasing roughness height, the saturation amplitude remains largely unaffected by changes in the roughness height. The presence of a large laminar extent of nonlinear saturation gives rise to a certain difficulty in using linear methods (such as $e^N$ or LPSE) to predict transition. The futility of such approaches is expressed by Arnal (1994) and Reed et al. (1996), who show that linear methods do not work in correlating transition for crossflow-dominated boundary layers.

**Modal Decomposition** Radeztsky et al. (1994) described a measurement technique that allows the experimentally obtained stationary crossflow structure to be decomposed into its spatial modes. Using a high-resolution traversing mechanism, hotwires are carefully moved through the boundary layer along a predetermined path. Data are acquired at numerous spanwise locations, from which modal information is extracted using a spatial power spectrum. Reibert et al. (1996) used a slightly modified technique to more objectively determine the modal content. Under certain conditions, the amplitude of the fundamental disturbance mode and eight harmonics are successfully extracted from the experimental data.

**Excitation of Less Unstable Modes** Using the modal decomposition technique described above, Reibert et al. (1996) investigated the effect of roughness-induced forcing at a wavelength three times that of the most unstable stationary mode (according to linear theory). A cascading of energy from the fundamental to higher modes (smaller wavelengths) was observed, leading to nonlinear interactions among the fundamental mode and its harmonics. Transition was observed to occur slightly earlier compared to forcing at the most unstable wavelength, and the saw-tooth transition front was much more “regular” in span. These data indicate that nonlinear interactions among multiple modes are important in determining the details of transition.

**Excitation of Subcritical Modes** Continuing the experiments by Reibert et al. (1996), Saric et al. (1998) described a set of experiments in which the stationary crossflow disturbance is forced with subcritical roughness spacing, i.e., the spacing between roughness elements is less than the wavelength of the most unstable mode. Under these conditions, the rapid growth of the forced mode completely suppresses the linearly most unstable mode, thereby delaying transition beyond its
“natural” location (i.e., where transition occurs in the absence of artificial roughness). These data demonstrate that surface roughness can be used to control the stationary crossflow disturbance wave-number spectrum in order to delay transition on swept wings.

**STRUCTURE IDENTIFICATION USING POD** Chapman et al. (1998) applied linear stochastic estimation and proper orthogonal decomposition (POD) to identify the spatio-temporal evolution of structures within a swept-wing boundary layer. Detailed measurements are acquired using multielement hotfilm, hotwire, and crosswire anemometry. These data allow the POD to objectively determine (based on energy) the modes characteristic of the measured flow. Data are acquired through the transition region, from which an objective transition-detection method is developed using the streamwise spatial POD solutions.

**CFD COMPARISONS (DNS)** DNS have historically been constrained by computer resources and algorithmic limitations; however, some successes have been achieved in relation to the stationary crossflow problem. Reed & Lin (1987) and Lin & Reed (1993) performed DNS for stationary waves on an infinite-span swept wing similar to the ASU experiments. Meyer & Kleiser (1990) investigated the disturbance interactions between stationary and traveling crossflow modes on a swept flat plate using Falkner-Scan-Cooke similarity profiles for the basic state. The results are compared to the experiments by Müller & Bippes (1989). With an appropriate initial disturbance field, the nonlinear development of stationary and traveling crossflow modes is simulated reasonably well up to transition. Wintergerste & Kleiser (1995) continued this work by using DNS to investigate the breakdown of crossflow vortices in the highly nonlinear final stages of transition.

**CFD COMPARISONS (PSE)** Combining the ability to include nonparallel and nonlinear effects with computationally efficient parabolic marching algorithms, the PSE developed by Herbert (1997a) have recently been used to successfully model the crossflow instability. For swept-wing flows, NPSE calculations exhibit the disturbance amplitude saturation characteristic of the DLR and ASU experiments. Wang et al. (1994) investigated both stationary and traveling crossflow waves for the swept airfoil used in the ASU experiments and predicted nonlinear amplitude saturation for both types of disturbances. It is suggested that the interaction between the stationary and traveling waves is an important aspect of the transition process.

Haynes & Reed (2000) recently validated the NPSE approach for 3-D flows subjected to crossflow disturbances. Here a detailed comparison of NPSE results with the experimental measurements by Reibert et al. (1996) show remarkably good agreement. Haynes & Reed (2000) independently computed the inviscid flow for the model, from which the edge boundary conditions were generated for the boundary-layer code. The initial conditions for the NPSE calculation (with curvature) were obtained by solving the local LST equations at 5% chord location for the fundamental stationary mode and adjusting its RMS amplitude such that the total disturbance amplitude matched that of the experiment at 10% chord. The
NPSE was then matched from 5% chord to 45% chord. Transition occurred on the experimental model at 47% chord. The primary and higher modes all grow rapidly at first and saturate at approximately 30% chord. This is due to a strong nonlinear interaction among all the modes over a large chordwise distance. An interesting discovery by Haynes & Reed (2000) is that the flow is hypersensitive to seemingly negligible streamline and body curvature and that inclusion of curvature terms is required to adequately match the experimental data. Figure 3 shows the comparison of the experimental $N$-factor (log of the amplitude ratio) with LPSE, NPSE, and LST as the crossflow vortex grows and then saturates downstream. All the computations include curvature. It is clear that the linear theories fail to accurately describe saturation for this situation.

Figure 4 shows a comparison of the experimental and computational total streamwise velocity contours at 45% chord; the agreement between the NPSE and the experiments is excellent. The computations in Figures 3 and 4 use only an initial amplitude. The basic state is computed independently from the experimental data.

There has been much debate about the effects of curvature. For the ASU configuration, the inclusion of curvature has a very small effect on the metric coefficients. The maximum values of metric coefficients occur at 5% chord where they are of
the order of $1 \times 10^{-2}$ and $10^{-3}$, respectively. They both drop off sharply with increasing chordwise distance. These values may compel the researcher to neglect curvature, but the work by Haynes & Reed (2000) demonstrates conclusively that small changes in the metric coefficients can have a significant effect on the development of crossflow vortices.

Radeztsky et al. (1994) studied the effects of angle-of-attack. Here, in a case of weak favorable pressure gradient, the experiments showed that the crossflow disturbance is decaying in disagreement with various linear theories (LST, LPSE/without curvature, and LST/with curvature) that predicted a growing disturbance. Radeztsky et al. (1994) concluded that the disagreement was due to nonlinearity. For this case, Haynes & Reed (2000) found that the LPSE/with curvature and NPSE/with curvature both agreed with the experiment, indicating that, in fact, the crossflow disturbance decays and that there is a strong sensitivity to changes in curvature, nonparallel effects, and pressure gradient (angle-of-attack). The disturbance was linear for this case.

3.4. Control with Distributed Roughness

Two important observations concerning the distributed roughness results by Reibert et al. (1996) are (a) unstable waves occur only at integer multiples of the primary disturbance wave number and (b) no subharmonic disturbances are destabilized. Spacing the roughness elements with wave number $k = 2\pi/\lambda$ apart excites harmonic disturbances with spanwise wave numbers of $2k, 3k, \ldots, nk$ (corresponding to $\lambda/2, \lambda/3, \ldots, \lambda/n$) but does not produce any unstable waves with “intermediate” wavelengths or with wavelengths greater than $\lambda$.

Following this lead, Saric et al. (1998) investigated the effects of distributed roughness whose primary disturbance wave number does not contain a harmonic at $\lambda_s = 12$ mm (the most unstable wavelength according to linear theory). By changing the fundamental disturbance wavelength (i.e., the roughness spacing) to 18 mm, the velocity contours clearly showed the presence of the 18-, 9-, and 6-mm wavelengths. However, the linearly most unstable disturbance ($\lambda_s = 12$ mm) has been completely suppressed. Moreover (and consistent with all previous results), no subharmonic disturbances are observed, which shows that an appropriately designed roughness configuration can, in fact, inhibit the growth of the (naturally occurring) most unstable disturbance. When the disturbance wavelength was forced at 8 mm, the growth of all disturbances of greater wavelength was suppressed. The most remarkable result obtained from the subcritical roughness spacing is the dramatic effect on transition location: In the absence of artificial roughness, transition occurs at 71% chord. Adding roughness with a spanwise spacing equal to the wavelength of the linearly most unstable wave moves transition forward to 47% chord. However, subcritical forcing at 8-mm spanwise spacing actually delays transition beyond the pressure minimum and well beyond 80% chord (the actual location was beyond view). This promising technique is currently being evaluated for supersonic flight (Saric & Reed 2002).
Subsequent to the experiments, the NPSE results (Haynes & Reed 2000) confirmed this effect. In a DNS solution, Wassermann & Kloker (2002) have shown the same stabilization due to subcritical forcing. Using the same independent approach in terms of the calculation of the basic state, they demonstrated the stabilization due to subcritical roughness and coined the name transition delay by “upstream flow deformation.”

3.5. Secondary Instabilities

Once stationary vortices reach saturation amplitude, this state can persist for a significant streamwise distance. The velocity contours of Figures 2 and 4 show low-momentum fluid above high-momentum fluid, which produces a double inflection point in the wall-normal velocity profile. There is also an inflection point in the spanwise profile. These inflection points are high in the boundary layer, and the saturated vortices become unstable to a high-frequency secondary instability that ultimately brings about transition to turbulence. Because of the importance of the secondary instabilities in determining the location of breakdown of the laminar flow, there have been a number of investigations, both experimental and computational, in this area. Bippes (1999) included details on the German efforts, in particular, the work by Lerche (1996) that emphasizes secondary instabilities in flows with higher turbulence levels and traveling crossflow waves. Boiko et al. (1995, 1999) covered recent efforts involving secondary instabilities in the Russian traveling-wave experiments.

Poll (1985) conducted the first crossflow experiment for which a high-frequency disturbance was observed prior to transition. Traveling crossflow waves were observed with a dominant frequency of 1.1 kHz for $Re_c = 0.9 \times 10^6$. Increasing the chord Reynolds number to $1.2 \times 10^6$ increased the traveling crossflow frequency to 1.5 kHz and also included an intermittent signal at 17.5 kHz superposed on the underlying traveling crossflow waves. Poll (1985) noted that increasing the Reynolds number beyond $1.2 \times 10^6$ resulted in turbulent flow at the measurement location, so the high-frequency signal appeared only in a narrow range just prior to transition. Poll (1985) attributed the existence of the high-frequency component to intermittent turbulence.

Kohama et al. (1991) investigated a high-frequency secondary instability specifically as a source of breakdown. This experiment combined hotwire measurements and flow visualizations and was intended to determine the location and behavior of the secondary instability mode relative to visualized breakdown patterns. The Kohama et al. (1991) experiments clearly show that there is a growing high-frequency mode in the region upstream of transition that can be associated with an inviscid instability of the distorted mean flow. However, a concern can be raised because the measurements were made without a well-controlled primary disturbance state. Experiments subsequent to this work used arrays of micron-sized roughness elements near the leading edge that established the spanwise uniformity both of the stationary vortex amplitudes and of the transition location. Without the benefit
of this technique, the data obtained by Kohama et al. (1991) likely spanned a wide range of stability behavior despite having been obtained at a single-chord position. Improvements in experimental techniques mean that more recent secondary instability experiments have replaced the work by Kohama et al. (1991) as the best source for secondary instability data.

Kohama et al. (1996) provided somewhat more detail than Kohama et al. (1991) by including velocity fluctuation maps that are filtered to give either primary instability or secondary instability fluctuation levels. Kohama et al. (1996) concluded that a “turbulent wedge starts from the middle height of the boundary layer” and that this behavior is different from the usual picture of a turbulent wedge that originates in the high-shear regions in naphthalene flow-visualization experiments. A subsequent swept-plate experiment by Kawakami et al. (1999) was conducted to further refine these measurements. This experiment featured a small speaker mounted flush with the surface that permitted tracking of particular secondary instability frequencies. Without acoustic forcing, two separate high-frequency bands of disturbances were observed to be unstable. At a chord Reynolds number of $4.9 \times 10^6$, a band located between 600 and 2.5 kHz destabilized just downstream of $x/c = 0.35$, and a second band located between 2.5 and 4.0 kHz destabilized just upstream of $x/c = 0.50$. Transition was observed around $x/c = 0.70$. With acoustic forcing applied, the secondary instability frequency with the largest growth between $x/c = 0.40$ and $x/c = 0.475$ was observed to be 1.5 kHz.

In an effort to provide a more concrete experimental database on the behavior of the secondary instability, White & Saric (2003) conducted a very detailed experiment that tracked the development of secondary instabilities on a swept wing throughout their development for various Reynolds numbers and roughness configurations. They found a number of unique secondary instability modes that can occur at different frequency bands and at different locations within the stationary vortex structure. In this experiment, as many as six distinct modes are observed between 2 and 20 kHz. The lowest-frequency mode is nearly always the highest amplitude of all the secondary instabilities and is always associated with an extremum in the spanwise gradient, $\partial U/\partial z$, which Malik et al. (1996, 1999) refer to as a mode-I or $z$ mode. Higher-frequency modes include both harmonics of the lowest-frequency $z$ mode that appear at the same location within the vortex and distinct mode-II or $y$ modes that form in the $\partial U/\partial y$ shear layer in the portion of the vortex farthest from the wall. The lowest-frequency mode is typically detected upstream of any of the higher-frequency modes. However, many higher-frequency modes appear within a very short distance downstream. All of the secondary instability modes are amplified at a much greater rate than the primary stationary vortices (even prior to their saturation). The rapid growth leads very quickly to the breakdown of laminar flow, within $\sim 5\%$ chord of where the secondary instability is first detected. A consequence of this for transition-prediction methodologies is that adequate engineering predictions of transition location could be obtained from simply identifying where the secondary instabilities are destabilized because they lead to turbulence in such a short distance downstream of their destabilization.
An interesting feature of the breakdown of the stationary vortex structure is that it is highly localized. Spectra obtained by White & Saric (2003) at various points within the structure indicate that the first point to feature a broad, flat velocity-fluctuation spectrum characteristic of turbulence is very close to the wall in the region of highest wall shear. Other points in the structure remain essentially laminar for some distance downstream of the initial breakdown location. This finding supports the notion of a turbulent wedge originating near the wall, in contrast to what Kohama et al. (1996) concluded.

A successful computational approach to the secondary instability was presented by Malik et al. (1994), who used an NPSE code to calculate the primary instability behavior of stationary disturbances of a swept Hiemenz flow. As described previously, the NPSE approach successfully captures the nonlinear effects including amplitude saturation. The distorted meanflow provides a basic state for a local, temporal secondary instability calculation. The most unstable frequency is approximately one order of magnitude greater than the most unstable primary traveling wave, similar to the results by Kohama et al. (1991), and the peak mode amplitude is “on top” of the stationary crossflow vortex structure. This location corresponds to what Malik et al. (1996) (see below) referred to as the mode-II secondary instability.

In order to obtain a more direct comparison to experimental data, Malik et al. (1996) used parameters designed to match the conditions found for the swept-cylinder experiment by Poll (1985) and the swept-wing experiment by Kohama et al. (1991). The calculations by Malik et al. (1996) revealed that the energy production for a mode-I instability is dominated by the term \( \langle u_2 w_2 \rangle \partial U_2 / \partial z_2 \) and the mode-II instability is dominated by \( \langle u_2 v_2 \rangle \partial U_2 / \partial y_2 \), where the subscript 2 refers to a primary-vortex-oriented coordinate system. This energy-production behavior suggests that the mode-I instability is generated primarily by inflection points in the spanwise direction and that the mode-II instability is generated by inflection points in the wall-normal direction. This situation is analogous to the secondary instabilities of Görtler vortices (Saric 1994). Malik et al. (1996) claimed that the fluctuations observed by Kohama et al. (1991) are mode-II instabilities, but the spectral data presented by Kohama et al. (1991) likely includes contributions of both the type-I and type-II modes. Although one or the other production mechanism may dominate for a particular mode, it is too simplistic to assume that only the spanwise or wall-normal inflection points are responsible for the appearance of a particular mode: with such a highly distorted 3-D boundary layer, all possible instabilities must be evaluated.

Malik et al. (1996) also computed the secondary instability behavior observed by Poll (1985) and predicted a 17.2-kHz mode compared to Poll’s high-frequency signal, which occurred at 17.5 kHz. Based on the shape of this disturbance, Malik et al. (1996) claimed that this is a type-II mode. Malik et al. (1999) applied the same approach to the swept-wing experiments by Reibert et al. (1996). Malik et al. (1999) again applied a local, temporal stability of the stationary crossflow vortices that are established by the primary instability and found that better transition correlation
results can be obtained by following the growth of the secondary instability in an $N$-factor calculation than by simply basing a prediction on the location at which the secondary instability destabilizes. A method based on the primary instability alone cannot adequately predict transition location.

Malik et al. (1999) found that a type-II mode becomes unstable upstream of any type-I mode. This is not what was observed in the experiment by White & Saric (2003) in which type-I modes always appeared upstream of type-II modes. This apparent difference is likely due to the fact that freestream disturbance levels in the experiment are stronger at frequencies near 3 kHz that correspond to the type-I modes than at the higher frequencies near 6 kHz that correspond to the type-II modes. The theory considers only growth rates and does not account for initial disturbance levels.

An alternative to the approach used by Malik et al. (1994, 1996, 1999) is presented by Koch et al. (2000), who found the nonlinear equilibrium solution of the primary flow. Koch et al. (2000) used the nonlinear equilibrium solution as a receptivity-independent basic state for a Floquet analysis of secondary instabilities of the saturated vortices. Yet another approach is by Janke & Balakumar (2000), who used an NPSE for the base flow and a Floquet analysis for the secondary instabilities. Both Koch et al. (2000) and Janke & Balakumar (2000) are in general agreement with the various computations by Malik et al. (1994, 1996, 1999).

Högberg & Henningson (1998) pursued a DNS approach to the problem of the stationary-vortex saturation and the ensuing secondary instability. These authors imposed an artificial random disturbance at a point where the stationary vortices are saturated. These disturbances enhance both the low- and high-frequency disturbances downstream, and each frequency band has a distinct spatial location, with the high-frequency disturbance located in the upper part of the boundary layer and the low-frequency disturbance located in the lower part. Spectral analysis of the resulting disturbance field shows that the most amplified high frequency is somewhat more than an order of magnitude higher frequency than the most amplified traveling primary disturbance. Another high-frequency peak at approximately twice this frequency is also evident in the spectra. This peak likely corresponds to a type-II mode, although this feature is not described by the authors.

Wassermann & Kloker (2002) present another highly resolved DNS study of nonlinear interactions of primary crossflow modes, their secondary instabilities, and eventual breakdown to turbulence. They emphasize disturbance wave packets that may be more realistic than single-mode disturbances. One of the most important findings obtained from the wave-packet approach is that unevenly spaced primary vortices of differing strengths can interact in such a way as to bring about an earlier onset of secondary instabilities and breakdown than would be found from a single-mode disturbance. Wassermann & Kloker (2002) also found that, when the forcing that initiates the high-frequency secondary instability in their simulation is removed, the secondary instability disturbances are convected downstream, out of the computational domain. This indicates that the secondary instability is convective and that the explosiveness of the growth of the secondary instability is
not associated with an absolute instability. The advantage of the DNS solution by Wassermann & Kloker (2002) is its ability to reveal the rather intimate details of the breakdown process. As such, their work is one of the foundation contributions.

To date, the various approaches to the secondary instability problem (experimental, NPSE, and DNS) have achieved rather remarkable agreement in terms of identifying the basic mechanisms of the secondary instability, unstable frequencies, mode shapes, and growth rates. A comparison of three of the most recent efforts is shown in Figure 5. This comparison shows agreement on the location of the breakdown and indicates that it is associated with an inflection point in the spanwise direction \( \partial U / \partial z \).

4. Swept Cylinder

The discussion of the crossflow instability in the preceding section pertains to boundary layers with characteristics typical of large portions of swept wings over which boundary layers develop in response to weakly favorable pressure gradient and low wall and streamline curvature. In contrast, boundary layers in the leading-edge region of swept wings can exhibit different stability characteristics than those observed farther downstream owing to the much stronger pressure gradient, wall curvature, and streamline curvature that exist near the leading edge. These flows are referred to as swept-cylinder flows to distinguish them from both swept-wing and attachment-line flows.

The swept-cylinder flow near the leading edge is distinct from the flow along the attachment line. Flow along the attachment line has stability characteristics different from swept-cylinder flow because of the various roles of surface and streamline curvature in the two configurations. Attachment-line stability and contamination are discussed at length by Reed & Saric (1989) but are outside of the scope of this review. Reed et al. (1996) and Theofilis (1998) discuss recent work concerning attachment-line issues.

For swept-cylinder flows, one of the principal recent findings is the discovery by Itoh (1994) that highly curved inviscid streamlines produce a centrifugal-type boundary-layer instability that is unique from the other 3-D boundary-layer instabilities that had been identified at the time of an earlier review by Reed & Saric (1989). Because the flow near the leading edge of swept cylinders features strongly curved streamlines, it may exhibit a streamline-curvature instability similar to that observed at small Reynolds numbers on rotating disks. Although it has yet to be demonstrated to have flight applicability, the streamline-curvature instability may play an important role under some set of conditions.

Itoh (1996) specifically examined the flow in the leading-edge region of swept cylinders. The key findings are that, for intermediate values of the inclination of the inviscid streamline to the chord direction (i.e., starting somewhat downstream of the leading edge), the streamline-curvature instability has the lowest critical Reynolds numbers of all instability modes and that the disturbances produced
Figure 5  Mode-I velocity fluctuation contours: (a) Figure 7 from Malik et al. (1999), (b) Figure 20b from Wassermann & Kloker (2002), and (c) Figure 11 from White & Saric (2003).
by the streamline-curvature instability take the form of streamwise vortices, and that these vortices are even more closely aligned with the inviscid stream than are crossflow vortices. Very close to the leading edge where the streamline inclination is large, crossflow waves are expected to be the most unstable disturbances. For a fixed value of the streamline inclination, linear-stability neutral curves for the swept-cylinder exhibit a two-lobed structure that suggests competition between crossflow and streamline-curvature instabilities, similar to what is observed for rotating-disk boundary layers. Itoh (1996) showed that the dynamics of this competition depend on sweep angle. As the sweep is increased the minimum critical Reynolds number for the streamline curvature instability is unchanged, but the point at which the critical Reynolds number of the streamline-curvature instability is less than that of the crossflow instability moves farther downstream, and this means that in the cases of high sweep, streamline curvature effects will be observed over a larger region than is typical of lower-sweep configurations. However, Itoh (1996) pointed out that this result is only valid close to the leading edge, so the results should not be taken to suggest that the streamline-curvature instability dominates farther downstream where the basic state deviates from a leading-edge-type flow.

Takagi & Itoh (1998) and Tokugawa et al. (1999) performed swept-cylinder experiments to verify the theoretical predictions by Itoh (1996) regarding the behavior of the streamline-curvature instability in the leading-edge region. Controlled point-source disturbances are introduced, and a developing disturbance wedge provides a means of separating crossflow-related and streamline-curvature-related disturbances. Itoh (1966) predicted that crossflow and streamline-curvature instabilities propagate in opposite directions, and Tokugawa et al. (1999) observed that, inside the disturbance wedge, disturbances with phase-distributions characteristic of the crossflow and streamline-curvature instabilities dominate in two separate disturbance-amplitude maxima that exist on either side of the wedge, providing confirmation of the general findings by Itoh (1996). More recently, Itoh (2000) included nonparallel effects, and Yokokawa et al. (2000) used a DNS model. The DNS results show that disturbances that propagate in the direction associated with the streamline-curvature instability are characterized by vorticity oriented primarily in the wall-normal direction in contrast with the previous analyses by Itoh (1995, 1996) that showed streamline-curvature disturbance modes are vortices oriented in the streamwise and not the wall-normal direction. This discrepancy has not been resolved.

5. CONCLUSIONS

Boundary-layer transition in 3-D flows is a complicated process involving complex geometries, multiple instability mechanisms, and nonlinear interactions. Yet significant progress has recently been made toward understanding the instability and transition characteristics of 3-D flows. In terms of the crossflow problem, the past decade has produced several important discoveries that include tools, such as
instrumentation that can be applied to the flight-test environment,
- POD methods to interpret wind-tunnel and flight-test transition data,
- validation with careful experiments of NPSE codes to predict all aspects of stationary disturbance growth.

Several important factors have also been identified:
- environmental conditions on the appearance of stationary and traveling waves,
- secondary instability causing local transition in stationary-crossflow-dominated flows,
- extreme sensitivity of the stationary disturbance to leading-edge, very small, surface roughness,
- nonlinear effects and modal interaction,
- extreme sensitivity of stationary-wave growth to very weak convex curvature.

One result has been a better understanding of rotating-disk behavior. In addition, by carefully studying the basic physics, these advances have led to the promising application of artificial roughness at the leading edge to control the crossflow instability and delay transition on swept wings in flight.

The study of 3-D boundary-layer instability still offers challenges to the fluid-mechanics community. Important factors such as receptivity—the process by which external disturbances enter the boundary layer and create the initial conditions for an instability—are still not completely understood. Yet in spite of this, careful experiments and companion accurate computations have resulted in significant progress toward understanding a difficult problem and offer promise of even further advances in the future.

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Figure 2: Velocity contours from Reibert et al. (1996).
Figure 4: Velocity contour comparisons.