ABSTRACT

Laminar-turbulent transition is highly initial- and operating-condition dependent. Finding careful, archival experiments for comparison is the main validation issue; few exist. The CFD formulations validated to date demonstrate that if the environment and operating conditions can be modelled and input correctly, the computations (nonlinear Parabolized Stability Equations and Direct Numerical Simulations) agree quantitatively with the experiments. A review of validation and verification, challenges of validation, and recent results for 3-D boundary layers and receptivity problems are presented.

REVIEW OF ROADMAP TO TRANSITION

Motivation

Transition location can be a significant source of uncertainty in the accurate prediction of aerodynamic forces (lift and drag) and losses and heating requirements for gas-turbine-engine blades and vanes, low-Reynolds-number vehicles, submarines and torpedoes, subsonic and supersonic civil transports and fighters, and hypersonic and re-entry vehicles. For example, in low-chord-Reynolds-number applications where laminar separation is important, a significant portion of the flow is transitional and it is therefore impossible to predict surface phenomena such as drag and heat transfer unless the flow in the transition region is understood. Being off by a factor of two in the location of the transition point can make a significant difference in these flows. For high-speed flows, Reed et al. (1997) discuss progress on issues such as instability studies, nose-bluntness and angle-of-attack effects, and leading-edge-contamination problems from theoretical, computational, and experimental points of view. Also included is a review of wind-tunnel and flight data, including high-Re flight transition data, the levels of noise in flight and in wind tunnels, and how noise levels can affect parametric trends.

When there is some knowledge of transition location, mean laminar and turbulent heat transfer can usually be computed to better than 25% accuracy. However laminar-turbulent transition in the boundary layer is usually estimated from crude algebraic correlations. The scatter in these correlations can be a factor of 2 or more. Because transition causes heat transfer to rise by a factor of about 5 in high-speed flows, this uncertainty in transition location dominates overall uncertainty in heat-transfer predictions. Clearly, knowledge of the transition process is crucial for accurate vehicle heating and drag predictions over the whole flight regime.

Also, the efficacy of transition control depends largely on the physics of the transition process. Whether one desires to delay transition by the various techniques of Laminar Flow Control (LFC; Pfenninger, 1965, 1977; Reshotko, 1984, 1985; Collier 1993; Joslin 1998) or to encourage transition for enhanced mixing or separation delay, the most effective strategy is to capitalize on the most unstable disturbances. Again, knowledge of the transition process is key to efficient application.

Of interest to the turbulence community is the fact that boundary-layer flows are open systems, strongly influenced by and dependent on freestream and wall conditions. Breakdown has been well documented to vary considerably when the operating conditions change (Saric and Thomas, 1984; Singer et al., 1989; Corke, 1990). The transition process then provides the vital upstream conditions from which the downstream turbulent flowfield evolves and it is reasonable to imagine that different transition patterns give rise to different turbulence characteristics downstream.

Introduction

Establishing the origins of turbulent flow and transition from laminar to turbulent flow remains an important challenge of fluid mechanics. The common thread connecting aerodynamic applications is the fact that they deal with bounded shear flows (boundary layers) in open systems (with different upstream or initial amplitude conditions). It is well known that the
stability, transition, and turbulent characteristics of bounded shear layers are fundamentally different from those of free shear layers (Morkovin 1969, Reshotko 1976; Bayley et al 1988). Likewise, open systems are fundamentally different from those of closed systems. The distinctions are trenchant and thus form separate areas of study.

At the present time no mathematical model exists that can predict the transition Reynolds number on a flat plate. One obvious reason for this is the variety of influences such as freestream turbulence, surface roughness, sound, etc. which are incompletely understood. A historical perspective of the progress and issues in transition prediction is given by Reshotko (1997). Reed et al (1996) review the linear-stability literature and discuss the importance of this work as it relates to transition and aircraft skin-friction reduction. With the maturation of linear-stability methods and the conclusions that breakdown mechanisms are initial-condition dependent (Saric & Thomas 1984; Singer et al 1989; Corke 1990), more emphasis is now placed on the understanding of the source of initial disturbances than on the details of the later stages of transition.

Figure 1. Roadmap to transition (Reshotko et al.)

Boundary-Layer Transition

The process of transition for boundary layers in external flows can be qualitatively described using Figure 1 and the following (albeit, oversimplified) scenario based on one of the different “roadmaps” to turbulence developed over the years (Morkovin et al 1994).

Disturbances in the freestream, such as sound or vorticity, enter the boundary layer as steady and/or unsteady fluctuations of the basic state. This part of the process is called receptivity (Morkovin 1969) and it establishes the initial conditions of disturbance amplitude, frequency, and phase for the breakdown of laminar flow (Saric et al. 2002). In Figure 1, the initial amplitude increases schematically from left to right. The convention for turbulence level is low $u' < 10^{-3}$, moderate $u' < 10^{-2}$, high $u' < 10^{-1}$, and very high $u' >10^1$. Initially these disturbances may be too small to measure and they are observed only after the onset of an instability. A number of different instabilities can occur independently or together and the appearance of any particular type of instability depends on Reynolds number, wall curvature, sweep, roughness, and initial conditions. If Figure 1 is entered with weak disturbances and path A is followed, the initial growth of these disturbances is described by linear stability theory of primary modes (i.e. linearized, unsteady, Navier-Stokes). For two-dimensional (2-D) boundary layers, this growth is weak, occurs over a long streamwise length scale, and can be modulated by pressure gradients, surface mass transfer, temperature gradients, etc. As the amplitude grows, 3-D and nonlinear interactions occur in the form of secondary instabilities. Disturbance growth is very rapid in this stage (now over a convective length scale) and breakdown to turbulence occurs.

Since the linear stability behavior can be calculated, transition prediction schemes often assume that transition follows path A and consider only the linear regime. This is justified on the assumption that external flows typically have weak freestream disturbances and the streamwise extent of the linear growth region is large compared to that of the nonlinear region. However, since the initial conditions (receptivity) are not generally known, only correlations between two systems with similar environmental conditions are possible. Recent critical reviews of these methods are found in Arnal (1994) and Reed et al (1996). In particular, for 3-D boundary layers (e.g. swept wings) and Görtler problems (concave surfaces), nonlinear distortions of the basic flow may occur early on due to the action of the primary instability. These flows are characterized by an extensive distance of nonlinear evolution with eventual saturation of the fundamental disturbance, leading to the strong amplification of very-high-frequency inflectional instabilities and breakdown (Saric 1994 and Saric et al. 2003). In this case linear stability theory does not accurately model disturbance growth.

At times, the freestream disturbances are so strong that the growth of linear disturbances is bypassed (Morkovin, 1969, 1993) and turbulent spots or
Historically, one had either path E in Figure 1 and while the phenomenon is not well understood it has been documented in cases of roughness and high freestream turbulence (Reshotko 1984, 1994, 2001). In this case, transition prediction schemes based on linear theory fail completely.

It is generally accepted that bypass refers to a transition process whose initial growth is not described by the primary modes of the Orr-Sommerfeld equation (OSE). Historically, one had either path A or E from which to choose as the road to turbulence. Recently however, considerable work in the area of transient growth has expanded our understanding of different paths by which transition to turbulence can occur.

Transient growth occurs when two, non-orthogonal, stable modes interact, undergo algebraic growth, and then decay exponentially. Streamwise vorticity and wall-normal vorticity appear to be important. This mechanism was first elucidated by Landahl (1980) and then Hultgren & Gustavsson (1981). The idea was used by Henningson et al (1993) and others. Recent reviews are given by Andersson et al (1999), Reshotko (2001), and Schmid & Henningson (2001).

Studies have shown that large amplitudes can be achieved through transient growth when the boundary layer is provided with appropriate initial conditions. Thus, the spectrum of initial conditions depends on receptivity. Returning to Figure 1, one can now say that, depending on amplitude, transient growth can lead to spanwise modulations of 2-D waves (path B), direct distortion of the basic state which leads to secondary or subcritical instabilities (path C), or direct bypass (path D).

In spite of progress, an overall theory remains rather incomplete with regard to predicting transition. Amplitude and spectral characteristics of the disturbances inside the laminar viscous layer strongly influence which type of transition occurs. Thus, it is necessary to understand how freestream disturbances are entrained into the boundary layer and create the initial amplitudes of unstable waves, i.e., to answer the question of receptivity.

Objectives

The three most widely used transition tools are linear stability theory (LST), parabolized stability equations (PSE), and direct numerical simulations (DNS). The next section describes the formulations of the various tools, with an emphasis on the very promising PSE.

Following are a review of validation and verification, challenges of validation, and recent results for 3-D boundary layers and receptivity problems. Herbert (1997), Reed et al. (1998), and Reed and Saric (2000) provide more examples.

FORMULATIONS

In this Section we present very brief descriptions of the three widely used numerical approaches for modern transition problems. Due to space limitations, the formulations of governing equations associated with LST and DNS are not presented here, but are detailed elsewhere. Our main focus is the stability and transition of boundary-layer-type flows.

For transition analysis, equations governing the disturbance are typically solved separately from the basic state. The basic-state formulations are not presented or discussed here, however the validity of these formulations must also be considered since the transition process is known to be sensitive to subtle changes in the basic state. The numerical accuracy of the basic state must be very high, because the stability and transition results will be very sensitive to small departures of the mean flow from its “exact” shape. The stability of the flow can depend on small variations of the boundary conditions for the basic state, such as freestream velocity or wall temperature. Therefore, basic-state boundary conditions must also be very accurate. See the discussion of Arnal (1994) and Malik (1990).

Linear Stability Theory

Linear stability theory has been the most widely used approximate method for stability analysis. See Reed et al. (1996) and Arnal (1994) for detailed formulations, applications, and limitations. In this approach the total flow is separated into a steady basic state and an unsteady disturbance. The basic state, by definition, satisfies the equations governing the total flow and represents the flow that exists in the absence of any environmental disturbances. This allows the basic-state and disturbance equations to be solved separately. For LST the basic state is approximated as “locally parallel” so that the wall-normal velocity component is set to zero and the remaining flow quantities are functions of the wall-normal direction only. The disturbances are assumed to be “small” enough to allow the nonlinear terms to be neglected. With these approximations and homogeneous boundary conditions, the disturbance equations feature coefficients which depend on the wall-normal coordinate only, and thus
the separation of variables into normal modes is possible and an eigenvalue problem results.

The state-of-the-art transition-prediction design tool involves linear stability theory coupled with an $e^N$ transition-prediction scheme and is applied at all speeds. The quantity $N$ is obtained by integrating the linear growth rate from the first neutral-stability point to a location somewhere downstream on the body, but $e^N$ represents nothing more than an amplitude ratio. The role of receptivity, not accounted for in linear stability theory, is key to the overall process as it defines the initial disturbance amplitude. Transition to turbulence will never be successfully understood or predicted without answering how freestream acoustic signals and turbulence enter the boundary layer and ultimately generate unstable waves. Clearly then, the study of receptivity promises significant advance in practical transition-prediction methods.

The basic design tool is the correlation of $N$ with transition Reynolds number for a variety of observations. The correlation will produce a number for $N$ (say 9) which is now used to predict transition Reynolds number for cases in which experimental data are not available.

**Parabolized Stability Equations**

In recent years the PSE have become a popular approach to stability analysis owing to their inclusion of nonparallel and nonlinear effects with relatively small additional resource requirements as compared with DNS (Herbert 1997). For linear PSE (LPSE), a single monochromatic wave is considered as the disturbance, which is decomposed into a rapidly varying “wave function” and a slowly varying “shape function”. Using a multiple-scales approach

$$
\phi'(x,y,z,t) = \overline{\phi}(\overline{x},y) \chi(x,z,t) + \text{c.c.} \quad (1)
$$

where

$$
\frac{\partial \chi}{\partial x} = i \alpha(\overline{x}), \quad \frac{\partial \chi}{\partial z} = i \beta, \quad \frac{\partial \chi}{\partial t} = -i \omega \quad (2)
$$

The “shape function” $\overline{\phi}$ and streamwise wavenumber $\alpha$ depend on the slowly varying scale $\overline{x} = x/R$ while the “wave function” $\chi$ depends on the rapidly varying scale $x$. The frequency is $\omega$ and the spanwise wavenumber is $\beta$. This gives the following form for the streamwise derivatives of disturbance quantities

$$
\frac{\partial \phi'}{\partial x} = \left\{ 1 - \frac{\partial \overline{\phi}}{R \partial \overline{x}} + i a \phi \right\} \chi + \text{c.c.}
$$

$$
\frac{\partial^2 \phi'}{\partial x^2} = \left\{ 1 - \frac{\partial^2 \overline{\phi}}{R \partial \overline{x}^2} + \frac{2i \alpha}{R} \frac{\partial \overline{\phi}}{\partial \overline{x}} + \frac{i \phi}{R} \frac{d \alpha}{dx} - \alpha^2 \right\} \chi + \text{c.c.} \quad (3)
$$

The explicit streamwise $O\left(\frac{1}{R^2}\right)$ second-derivative term is neglected. This yields the following system of equations

$$
\left( L_0 + L_1 \right) \overline{\phi} + L_2 \frac{\partial \overline{\phi}}{\partial \overline{x}} + \phi L_3 \frac{d \alpha}{dx} = 0 \quad (4)
$$

Here $L_0$ is the Orr-Sommerfeld operator, $L_1$ contains the nonparallel basic-state terms, and $L_2$ and $L_3$ arise due to the nonparallel disturbance terms.

The resulting system of equations is parabolic, so to complete the formulation, upstream (initial) and boundary conditions must be specified. The disturbance quantities are zero at the wall and as $y \to \infty$. If the analysis begins in a region where the initial disturbance amplitudes are small, linear stability theory can be used to obtain these initial conditions.

There still remains the matter of the ambiguity in streamwise dependence; applying a normalization condition ensures that any rapid changes in the streamwise direction will be “absorbed” by the wave function so that the shape function will vary slowly in this direction. For example, Haynes and Reed (2000) suggest the integral normalization

$$
\rho = \int_{y=0}^{\infty} \frac{\partial \overline{u}}{\partial x} \, dy = 0 \quad (5)
$$

Assuming the solution is known at streamwise location $x^i$, Haynes & Reed (2000) suggest the following streamwise marching algorithm:

Guess $\alpha \left( x^{i+1} \right)$

Solve equation 1 for $\phi^{i+1}$

Use $\phi^{i+1}$ to compute the error $\rho$
Use Newton’s method with $\phi^{i+1}$ to update $\alpha^{i+1}$.

Repeat steps 2-4 until $\rho$ is less than some tolerance.

The nonlinear PSE (NPSE) are derived in a fashion similar to LPSE with the exception that each disturbance quantity is transformed spectrally in the spanwise and temporal directions.

$$\phi'(x,y,z,t) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \tilde{\phi}_{n,k}(\bar{x},\bar{y}) \ast \text{shapefunction}$$

$$A_{n,k}(x)e^{i(k\beta_0 z-n\omega_0 t)}$$

$$dA_{n,k}(x)/dx = A_{n,k}i\alpha_{n,k}(x)$$

(6)

where

$$\frac{dA_{n,k}}{dx} = A_{n,k}i\alpha_{n,k}(x)$$

(7)

Here each mode $(n,k)$ is the product of a “shape function” and a “wave function”. The resulting system of equations is

$$\sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left\{ L_0 + L_1 \tilde{\phi} + L_2 \frac{\partial \tilde{\phi}}{\partial \bar{x}} + L_3 \frac{d\alpha}{d\bar{x}} \right\}_{n,k} \ast$$

$$A_{n,k}e^{i(k\beta_0 z-n\omega_0 t)} = N$$

(8)

The operators $L_i$ ($i = 0, 3$) assume the same meaning as in the LPSE form except that they are applied to each particular mode $(n,k)$, where $\alpha_0$ and $\beta_0$ are the fundamental frequency and spanwise wavenumber, respectively. During the marching procedure, each mode must individually satisfy the normalization condition.

The PSE formulation here utilizes a body-intrinsic coordinate system and the curvature is included in the associated metric coefficients. The marching procedure naturally aligns the disturbance wave propagation in the proper direction. The local radius of curvature of the wing appears in the equations through the following terms:

$$k_1 = 1 + \frac{y}{R_c}, k_2 = \frac{1}{y+R_c},$$

$$k_3 = -\frac{1}{R_c} \frac{dR_c}{dx} \approx 0,$$

$$k_4 = -\frac{y}{R_c^2} \frac{dR_c}{dx} \approx 0$$

(9)

where $R_c$ is the local dimensionless radius of curvature of the airfoil taken as positive or negative for convex or concave regions, respectively. For all the computations presented here, curvature is neglected in the basic-state analysis. This is because the basic-state curvature terms are the same order as the terms neglected according to the boundary-layer approximation so it would be inconsistent to retain them. In the limit of infinite curvature (flat plate), $R_c \rightarrow \infty$, so $k_1 = 1$ and $k_2 = 0$ are used in the stability equations for cases where curvature is neglected and the above equations are used for cases where curvature is retained.

Haynes and Reed (2000) discretized the PSE using 4th-order-accurate finite differences in the normal direction and up to 2nd-order-accurate backward differences in the streamwise direction.

The code LASTRAC (Chang 2003) has been developed at NASA Langley and is available with limited distribution.

**Direct Numerical Simulations**

Direct numerical simulations (DNS) play an important role in the investigation of transition. This trend will continue as considerable progress is made in the development of new, extremely powerful, parallel computers and numerical algorithms. In such simulations, the full Navier-Stokes equations are solved directly by employing numerical methods, such as finite-difference, finite-element, finite-volume, or spectral methods. An excellent review was that of Kleiser & Zang (1991) and the AGARD lecture on spatial simulations by Reed (1994) serves as a complementary companion to that of Kleiser (1993) on temporal simulations. The intent of these notes is to describe applications of spatial simulations and significant results obtained using this technique.

Transition is a spatially evolving process and the spatial DNS approach is widely applicable since it avoids many of the restrictions that usually have to be imposed.
in other models and is the closest to mimicking experiments. For example, no restrictions with respect to the form or amplitude of the disturbances have to be imposed, because no linearizations or special assumptions concerning the disturbances have to be made. Furthermore, this approach allows the realistic treatment of space-amplifying and -evolving disturbances as observed in laboratory experiments. The temporal simulation, by contrast, uses periodic boundary conditions in the streamwise direction (identical inflow and outflow conditions) and follows the time evolution of a disturbance as it convects through the flow; upstream influence is limited by this assumption. Moreover, in temporal simulations, the basic state is assumed to be strictly parallel, that is, invariant with respect to the streamwise coordinate. All of these restrictions noted are especially suspect when considering complex geometries, 3-D boundary layers, receptivity, and control. Fasel (1990) makes similar points. Kleiser (1993) discusses the issue of rapid growth near breakdown.

The basic idea of the spatial simulation is to disturb an established basic flow by forced, time-dependent perturbations. Then the reaction of this flow, that is, the temporal and spatial development of the perturbations, is determined by the numerical solution of the complete Navier-Stokes equations.

Problems associated with this method which preclude it from being used routinely for design include:

(a) A large amount of computer resources (cpu and memory) is usually required for solution. Because of the long fetch from the onset of instability to breakdown and the large amplitude ratios associated with this process \([O(e^{10})\] and larger\), resolution and bit accuracy limit how far into breakdown that a spatial simulation can go. Because of the large differences in amplitudes throughout the domain and the large growth rates known to exist near breakdown where smaller scales appear, truncation and round-off errors can easily contaminate the solution.

(b) There is a need to impose a nonintrusive downstream boundary condition since the periodic assumption (associated with temporal simulations) is no longer used. Several ideas are presented in these notes, however, there is typically a region of waste where the Navier-Stokes equations are not valid and the solution is discarded. In light of the discussion in (a), this adds to the resource problem.

(c) The use of either the Parabolized Stability Equations (PSE; Herbert 1997) or DNS simulations, both of which account for nonlinear and nonparallel effects, is hampered by our current lack of knowledge of the connection between the freestream and the boundary-layer response. A physically appropriate upstream or inflow condition must be specified. Efforts to bridge this gap are described in the section on receptivity.

Validation and Verification

Here we distinguish between verification and validation. Per the designations of Roache (1997), we consider verification to mean “confirming the accuracy and correctness of the code” (i.e. is the grid resolved, are there any programming errors in the codes, etc.). Validation requires verification of the code in addition to confirming the adequacy of the equations used to model the physical problem. Strictly speaking, a code can only be validated by comparison with quality experimental data. AIAA Guide 1998 should be consulted.

There are mainly three sources of error in the abstraction of continuous PDE's to a set of discrete algebraic equations; (1) discretization errors, (2) programming errors (bugs), and (3) computer round-off errors. The objective of code verification is then to completely eliminate programming errors and confirm that the accuracy of the discretization used in solving the continuous problem lies within some acceptable tolerance. Aside from specifying single or double precision, the code developer has little control over the computer round-off errors, but this is usually several orders of magnitude smaller than the discretization error and far less than the desired accuracy of the solution.

In this section we address programming and discretization errors. Many methods are discussed in the literature for code verification using grid refinement, comparison with simplified analytical cases, etc. For recent discussions see Roache (1997) and Oberkampf et al. (1995). Specific suggestions for testing a CFD code for the study of transition include (a) grid-refinement studies, (b) solving test problems for which the solution is known, (c) changing the “far-field” boundary locations systematically and re-solving, (d) comparing linear growth rates, neutral points, and eigenfunctions with linear stability theory, (e) running the unsteady code with time-independent boundary conditions to ensure that the calculations remain steady, and (f) running geometrically unsymmetric codes with symmetric conditions.

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In addition to the usual code verification techniques, there is a general method to verify the discretizations and locate programming errors by comparison with “manufactured” analytical solutions (Steinberg and Roache, 1985). This method is general in that it can be applied to any system of equations. Although it is an extremely powerful tool, this method has received relatively little attention in the literature. For clarity the technique is demonstrated on the Poisson equation.

\[
Lu = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = F(x, y) \quad (10)
\]

To solve this problem, discretize the operator \(L\) using some appropriate approximation (finite differences, spectral, etc.). In general, the exact solution is not available. Therefore, for verification purposes, force the solution to be some combination of analytical functions with nontrivial derivatives. For example, consider the system \( g = Lv = 5e^{3y} \sin(2x) \), which has an analytical solution \( v = e^{3y} \sin(2x) \). The exact solution can then be compared with the computed solution. Of course, manufactured solutions should be chosen with topological qualities similar to those anticipated for the solution to the “real” problem (e.g. gradients close to the wall). Proper choice for the manufactured solutions also allows the discretization of the boundary conditions to be verified. For large systems of equations a symbol manipulator is recommended for computing \( g \). If a bug occurs, zeroing the coefficients of some terms in equation (22) can help to isolate the bug.

**STATUS AND CHALLENGES OF VALIDATION**

Validation is defined as encompassing verification of the code as well as confirming that the equations used to model the physical situation are appropriate. The basis of validation is assumed to be a successful comparison with the few careful, archival experiments available in the literature. The next sections describe recent results for 3-D boundary layers and receptivity problems. To date the PSE have been applied to a variety of 2- and 3-D flow situations and are generally regarded as appropriate for convectively unstable flows. The reader is encouraged to consult the reviews of Herbert (1997) and Reed et al. (1998) for many more examples, as well as the contrasts with direct numerical simulations (DNS) and LST. For leading-edge receptivity problems, DNS will be demonstrated to have been successfully validated.

**3-D Boundary Layers**
have no effect on streamwise disturbances (Radeztsky et al., 1993). 3) Disturbance development can be dominated by nonlinearities during a large part of the transition process and the use of linear theory up to breakdown is inappropriate and can overestimate wave amplitude (Reibert et al., 1996; Haynes and Reed, 2000).

Whereas linear stability theory predicts that the travelling crossflow waves are more amplified than the stationary crossflow waves, many experiments observe stationary waves. The question of whether one observes stationary or travelling crossflow waves is cast inside the receptivity problem. Deyhle and Bippes (1996) and Bippes (1997) describe a series of comparative experiments in a low-turbulence and a high-turbulence tunnel. Their results show that travelling crossflow waves are observed in the high-turbulence tunnel rich in unsteady freestream disturbances and the dominant structure in a low-turbulence tunnel is a stationary crossflow vortex. Since the flight environment is more benign than the wind tunnel, one expects the low-turbulence results to be more important.

One of the important results to come out of the DLR group is the set of data that show early saturation of the disturbance amplitude and the failure of linear theory to predict the growth of the instability. They also report distorted mean profiles similar to those of Kohama et al. (1991) and Malik et al. (1994) due to the presence of the stationary corotating vortices. A similarity between the DLR and ASU experiments is the high N-factors and high amplitude of the mean-flow distortion (10%-20%). It is not surprising that linear theory fails.

For low-amplitude crossflow waves, Radeztsky et al. (1994) find that linear stability theory correctly predicts the expected wavelengths and mode shapes for stationary crossflow. For this case, Haynes and Reed (2000) find that linear theory including curvature correctly predicts the growth rates. As discussed in the following Section, this is not the case for higher-amplitude crossflow and the results of Reibert et al. (1996) and Haynes & Reed (2000) demonstrate conclusively that a nonlinear calculation is required to obtain complete agreement. This is shown in the next section.

**Parabolized Stability Equations** The NPSE approach has recently been validated for 3-D flows subjected to crossflow disturbances by Haynes & Reed (2000). Here a detailed comparison of NPSE results with the experimental measurements of Reibert et al. (1996) show remarkably good agreement. The configuration is an NLF(2)-0415 45°-swept airfoil at 4° angle-of-attack, so chosen to provide an extensive region of crossflow (at least back to mid-chord) for detailed study of the physics. A spanwise array of roughness elements is used near the airfoil leading edge (at 2% chord) to introduce spanwise-periodic crossflow disturbances into the boundary layer. According to LST, a spanwise spacing of 12 mm corresponds to the most highly amplified stationary crossflow disturbance. The walls of the ASU Unsteady Wind Tunnel were shaped to achieve a spanwise-independent basic-state flow – an “infinite wing” in CFD terms. The freestream turbulence are well documented to be \( O(10^{-4}) \) so that, with any surface roughness, stationary crossflow is expected. Reibert et al. (1996) provide all the details for the experimental facility and set-up.

Haynes and Reed (2000) used a panel-method code to compute the inviscid flow, from which the edge boundary conditions were generated for the boundary-layer code. Agreement between the experimental and computational \( C_p \) distribution is good.

As a baseline case to study the evolution of crossflow vortices, roughness elements with a spanwise spacing of 12 mm were placed on the experimental model. The initial conditions for the NPSE calculation (with curvature) were obtained by solving the local LST equations at 5% chord location for the fundamental [mode (0,1)] and adjusting its RMS amplitude such that the total disturbance amplitude matched that of the experiment at 10% chord. The NPSE was then marched from 5% chord to 45% chord. Transition occurred on the experimental model at 52% chord.

The primary and higher modes all grow rapidly at first and saturate at about 30% chord. This is due to a strong nonlinear interaction among all the modes over a large chordwise distance. From about 35% chord on, there is still strong nonlinear interaction among the primary and second harmonic, but not the others. The development of crossflow occurs in two stages. The first stage is linear and is characterized by small vertical \( v \) and spanwise \( w \) disturbance velocities convecting low-momentum fluid away from the wall and high-momentum fluid toward the wall. This exchange of momentum occurs in a region very close to the wall where there are large vertical gradients in the basic-state streamwise velocity. Because of this large gradient, the small displacements caused by the \( v \) and \( w \) disturbance components quickly lead to large disturbances \( u \) superposed on the basic state further downstream. This \( u \) component soon becomes too large and nonlinear interactions must be included in any calculations. This is the second stage, evidenced by
A roll-over seen in the streamwise-velocity contours. Figure 2 shows a comparison of the experimental and computational total streamwise velocity contours at 45% chord; the agreement between the NPSE and the experiments is excellent. Figure 3 shows the comparison of the experimental $N$-factor curves with LPSE, NPSE, and LST. The NPSE results include curvature. It is clear that the linear theories fail to accurately describe the transitional flow for this situation.

There has been much debate about the effects of curvature. For this configuration, the inclusion of curvature has a very small effect on the metric coefficients. The maximum values of $k_1$ and $k_2$ occur at 5% chord where they are the order of $1.01 \times 10^{-3}$, respectively. They both drop off sharply with increasing chordwise distance. These values may compel the researcher to neglect curvature, but the work of Haynes and Reed (2000) demonstrates conclusively that small changes in the metric coefficients can have a significant effect on the development of crossflow vortices.

Radeztsky et al. (1994) studied the effects of angle-of-attack (AOA). Here, in a case of weak favorable pressure gradient, the experiments showed that the crossflow disturbance is decaying in disagreement with various linear theories (LST, LPSE/without curvature, and LST/with curvature) that predicted the disturbance to be growing. Radeztsky et al. (1994) concluded that the disagreement was due to nonlinearity. For this case, Haynes and Reed (2000) found that the LPSE/with curvature and NPSE/with curvature both agreed with the experiment, indicating that in fact the crossflow disturbance decays and there is a strong sensitivity to changes in curvature, nonparallel effects, and pressure gradient (AOA). The disturbance was linear for this case.

Saric et al. (1998) observed that unstable waves occur only at integer multiples of the primary disturbance and no subharmonic disturbances are destabilized. They investigated the effects of distributed roughness whose primary disturbance wavenumber does not contain a harmonic at $12$ mm (the most unstable wavelength according to linear theory). In the absence of artificial roughness, transition occurs at $71\%$ chord. Adding roughness with a spanwise spacing equal to the wavelength of the linearly most unstable wave moves transition forward to $52\%$ chord. However, subcritical forcing at $8$ mm spanwise spacing actually delays transition beyond the pressure minimum and onto the trailing-edge flap at $80\%$ chord. The NPSE results confirmed this effect.

**Control with Distributed Roughness** Two important observations concerning the distributed roughness results of Reibert et al. (1996) are: (1) unstable waves occur only at integer multiples of the primary disturbance wavenumber; (2) no subharmonic disturbances are destabilized. Spacing the roughness elements with wavenumber $k = 2\pi/\lambda$ apart, excites harmonic disturbances with spanwise wavenumbers of $2k, 3k, \cdots, nk$ (corresponding to $\lambda/2, \lambda/3, \cdots, \lambda/n$) but does not produce any unstable waves with “intermediate” wavelengths or with wavelengths greater than $\lambda$.

Following this lead, Saric et al. (1998) investigate the effects of distributed roughness whose primary disturbance wavenumber does not contain a harmonic at $\lambda_s = 12$ mm (the most unstable wavelength according to linear theory). By changing the fundamental disturbance wavelength (i.e., the roughness spacing) to $18$ mm, the velocity contours clearly showed the presence of the $18$ mm, $9$ mm, and $6$ mm wavelengths. However, the linearly most unstable disturbance ($\lambda_s = 12$ mm) has been completely suppressed. Moreover (and consistent with all previous results), no subharmonic disturbances are observed. This shows that an appropriately designed roughness configuration can, in fact, inhibit the growth of the (naturally occurring) most-unstable disturbance. When the disturbance wavelength was forced at $8$ mm, the growth of all disturbances of greater wavelength was suppressed. The most remarkable result obtained from the subcritical roughness spacing is the dramatic affect on transition location: In the absence of artificial roughness, transition occurs at $71\%$ chord. Adding roughness with a spanwise spacing equal to the wavelength of the linearly most unstable wave moves transition forward to $47\%$ chord. However, subcritical forcing at $8$ mm spanwise spacing actually delays transition beyond the pressure minimum and well beyond $80\%$ chord (the actual location was beyond view). This promising technique is currently being evaluated for supersonic flight (Saric & Reed 2002).

Subsequent to the experiments, the NPSE results (Haynes & Reed 2000) confirmed this effect. In a DNS solution, Wassermann & Kloker (2002) have shown the same stabilization due to subcritical forcing. Using the same independent approach regarding the calculation of the basic state, they demonstrated the stabilization due to subcritical roughness and coined the name transition delay by “upstream flow deformation.”
Secondary Instabilities

Once stationary vortices reach saturation amplitude, this state can persist for a very significant streamwise distance. The velocity contours show low-momentum fluid above high-momentum fluid which produces a double inflection point in the wall-normal velocity profile. There is also an inflection point in the spanwise profile. These inflection points are high in the boundary layer and the saturated vortices become unstable to a high-frequency secondary instability that ultimately brings about transition to turbulence. Because of the importance of the secondary instabilities in determining the location of breakdown of the laminar flow, there have been a number of investigations, both experimental and computational, in this area. Bippes (1999) includes details on the German efforts, in particular, the work by Lerche (1996) and Lerche & Bippes (1995) that emphasizes secondary instabilities in flows with higher turbulence levels and traveling crossflow waves. Recent efforts involving secondary instabilities in the Russian traveling wave experiments are covered by Boiko et al. (1995, 1999).

The first crossflow experiment for which a high-frequency disturbance was observed prior to transition was by Poll (1985). Traveling crossflow waves were observed with a dominant frequency of 1.1 kHz for Reₐ = 0.9 × 10⁶. Increasing the chord Reynolds numbers to 1.2 × 10⁶ increased the traveling crossflow frequency to 1.5 kHz and also included an intermittent signal at 17.5 kHz superposed on the underlying traveling crossflow waves. Poll noted that increasing the Reynolds number beyond 1.2 × 10⁶ resulted in turbulent flow at the measurement location, so the high-frequency signal appeared only in a narrow range just prior to transition. Poll attributed the existence of the high-frequency component to intermittent turbulence.

A high-frequency secondary instability was specifically investigated as a source of breakdown by Kohama et al. (1991). This experiment combined hotwire measurements and flow visualizations and was intended to determine the location and behavior of the secondary instability mode relative to visualized breakdown patterns. It is clear from the Kohama et al. (1991) experiments that there is a growing high-frequency mode in the region upstream of transition that can be associated with an inviscid instability of the distorted mean flow. However, a concern can be raised because the measurements were made without a well-controlled primary disturbance state. Experiments subsequent to this work used arrays of micron-sized roughness elements near the leading edge that established the spanwise uniformity both of the stationary vortex amplitudes and the transition location. Without the benefit of this technique, the data obtained by Kohama et al. (1991) likely spanned a wide range of stability behavior despite having been obtained at a single chord position. Improvements in experimental techniques mean that more recent secondary instability experiments have replaced the work by Kohama et al. (1991) as the best source for secondary instability data. Kohama et al. (1996) provide somewhat more detail than Kohama et al. (1991) by including velocity fluctuation maps that are filtered to give either primary instability or secondary instability fluctuation levels. Kohama et al. (1996) conclude that a “turbulent wedge starts from the middle height of the boundary layer” and that this behavior is different from the usual picture of a turbulent wedge that originates in the high-shear regions in naphthalene flow-visualization experiments. A subsequent swept plate experiment by Kawakami et al. (1999) was conducted to further refine these measurements. Kawakami et al.’s experiment featured a small speaker mounted flush with the surface that permitted tracking of particular secondary-instability frequencies. Without acoustic forcing, two separate high-frequency bands of disturbances were observed to be unstable. At a chord Reynolds number of 4.9 × 10⁶, a band located between 600 Hz and 2.5 kHz destabilized just downstream of x/c = 0.35 and a second band located between 2.5 and 4.0 kHz destabilized just upstream of x/c = 0.50. Transition was observed around x/c = 0.70. With acoustic forcing applied, the secondary instability frequency with the largest growth between x/c = 0.40 and x/c = 0.475 was observed to be 1.5 kHz.

In an effort to provide a more concrete experimental database on the behavior of the secondary instability, White & Saric (2002) conduct a very detailed experiment that tracks the development of secondary instabilities on a swept wing throughout their development for various Reynolds numbers and roughness configurations. They found a number of unique secondary instability modes that can occur at different frequency bands and at different locations within the stationary vortex structure. In White & Saric’s experiment as many as six distinct modes are observed between 2 and 20 kHz. The lowest-frequency mode is nearly always the highest amplitude of all the secondary instabilities and is always associated with an extremum in the spanwise gradient, ∂U/∂z which Malik et al. (1996, 1999) refer to as a mode-I or z mode. Higher frequency modes include both harmonics of the lowest-frequency z mode that appear at the same location within the vortex and also distinct mode-II or y modes that form in the ∂U/∂y shear layer in the portion of the vortex farthest from the wall. The lowest frequency mode is typically detected upstream of any
of the higher frequency modes. However, many higher frequency modes appear within a very short distance downstream. All of the secondary instability modes are amplified at a much greater rate than the primary stationary vortices (even prior to their saturation). The rapid growth leads very quickly to the breakdown of laminar flow, within about 5% chord of where the secondary instability is first detected. A consequence of this for transition-prediction methodologies is that adequate engineering predictions of transition location could be obtained from simply identifying where the secondary instabilities are destabilized because they lead to turbulence in such a short distance downstream of their destabilization location. An interesting feature of the breakdown of the stationary vortex structure is that it is highly localized. Spectra obtained by White & Saric at various points within the structure indicate that the first point to feature a broad, flat velocity-fluctuation spectrum characteristic of turbulence is a point very close to the wall in the region of highest wall shear. Other points in the structure remain essentially laminar for some distance downstream of the initial breakdown location. This finding supports the notion of a turbulent wedge originating near the wall, not what was concluded by Kohama et al (1996).

A successful computational approach to the secondary instability was presented by Malik et al (1994) who used a NPSE code to calculate the primary instability behavior of stationary disturbances of a swept Hiemenz flow. As described previously, the NPSE approach successfully captures the nonlinear effects including amplitude saturation. The distorted meanflow provides a basic state for a local, temporal secondary instability calculation. The most unstable frequency is approximately one order of magnitude greater than the most unstable primary traveling wave similar to Kohama et al (1991) and the peak mode amplitude is “on top” of the stationary crossflow vortex structure. This location corresponds to what will be referred to below by Malik et al (1996) as the mode-II secondary instability.

In order to obtain a more direct comparison to experimental data, Malik et al (1996) used parameters designed to match the conditions found for the swept-cylinder experiment of Poll (1985) and the swept-wing experiment of Kohama et al (1991). The calculations of Malik et al (1996) reveal that the energy production for a mode-I instability is dominated by the term \( \langle u_2'w_2' \rangle \partial U_2/\partial z_2 \) and the mode-II instability is dominated by \( \langle u_2'v_2' \rangle \partial U_2/\partial z_2 \) where the subscript “2” refers to a primary-vortex-oriented coordinate system. This energy-production behavior suggests that the mode-I instability is generated primarily by inflection points in the spanwise direction and the mode-II instability is generated by inflection points in the wall-normal direction. This situation is analogous to the secondary instabilities of Görtler vortices (Saric 1994). Malik et al (1996) claim that the fluctuations observed by Kohama et al (1991) are mode-II instabilities but the spectral data presented by Kohama et al (1991) likely includes contributions of both the type-I and type-II modes. Although one or the other production mechanism may dominate for a particular mode, it is too simplistic to assume that only the spanwise or wall-normal inflection points are responsible for the appearance of a particular mode; with such a highly distorted 3-D boundary layer, all possible instabilities must be evaluated.

Malik et al (1996) also compute the secondary instability behavior observed by Poll (1985) and predict a 17.2-kHz mode compared to Poll’s high-frequency signal occurred at 17.5 kHz. Based on the shape of this disturbance, Malik et al claim that this is a type-II mode. The same approach is applied by Malik et al (1999) to the swept wing experiments of Reibert et al (1996). Malik et al (1999) again apply a local, temporal stability of the stationary crossflow vortices that are established by the primary instability and find that better transition correlation results can be obtained by following the growth of the secondary instability in an \( N \)-factor calculation than simply basing a prediction on the location at which the secondary instability destabilizes. A method based on the primary instability alone cannot adequately predict transition location.


A DNS approach to the problem of the stationary-vortex saturation and the ensuing secondary instability was pursued by Högberg & Henningson (1998). These authors impose an artificial random disturbance at a point where the stationary vortices are saturated. These disturbances enhance both the low- and high-frequency disturbances downstream, and each frequency band has a distinct spatial location, with the high-frequency disturbance located in the upper part of the boundary layer and the low-frequency disturbance located in the
lower part. Spectral analysis of the resulting disturbance field shows that the most-amplified high frequency is somewhat more than an order of magnitude higher frequency than the most-amplified traveling primary disturbance. Another high-frequency peak at approximately twice this frequency is also evident in the spectra. This peak likely corresponds to a type-II mode, although this feature is not described by the authors.

Another very highly resolved DNS study of nonlinear interactions of primary crossflow modes, their secondary instabilities, and eventual breakdown to turbulence is by Wassermann & Kloker (2002). Wassermann & Kloker emphasize disturbance wave packets that may be more realistic than single-mode disturbances. One of the most important findings obtained from the wave-packet approach is that unevenly spaced primary vortices of differing strengths can interact in such a way to bring about an earlier onset of secondary instabilities and breakdown than would be found from a single-mode disturbance. Also, Wassermann & Kloker find that when the forcing that initiates the high-frequency secondary instability in their simulation is removed, the secondary-instability disturbances are convected downstream, out of the computational domain. This indicates that the secondary instability is convective and that the explosiveness of the secondary instability’s growth is not associated with an absolute instability. The advantage of Wasserman & Kloker’s DNS solution is its ability to reveal the rather intimate details of the breakdown process. As such, the work is one of the foundation contributions.

At this time, the various approaches to the secondary instability problem, experimental, nonlinear PSE, and DNS, have achieved rather remarkable agreement in terms of identifying the basic mechanisms of the secondary instability, unstable frequencies, mode shapes, and growth rates. A comparison of three of the most recent efforts is shown in Figure 4. This comparison shows agreement on the location of the breakdown and that it is associated with an inflection point in the spanwise direction (an extremum in $\partial U/\partial z$).

**Challenges** These results validate the NPSE approach with curvature for 3-D crossflow-dominated transition cases where the disturbance inputs are known or controlled. Unfortunately, the disturbance inputs for flight conditions are not known. However, Schrauf et al. (1995) point out that transition information can be obtained by comparing results using initial conditions from “standard environments” to perform trade-off analyses. The NPSE has shown very encouraging results in validating against the available experimental databases, but more work is still needed to simulate physical initial conditions.

**Receptivity**

In spite of progress, an overall theory remains rather incomplete with regard to predicting transition. Amplitude and spectral characteristics of the disturbances inside the laminar viscous layer strongly influence which type of transition occurs. Thus, it is necessary to understand how freestream disturbances are entrained into the boundary layer and create the initial amplitudes of unstable waves, i.e., to answer the question of receptivity.

For the discussion here, the wave instability is assumed to be of the Tollmien-Schlichting (T-S) type in 2-D boundary layers and the ultimate goal is to determine how the wave amplitude within the boundary layer can be found from a freestream measurement. Even though the restriction to T-S waves may make this goal seem too modest, one soon finds sufficient challenges. Since receptivity deals with the generation rather than the evolution of instability waves in a boundary layer, neither departures from the linear-mode scenario nor details of the transition process itself are discussed here.

For incompressible flows, receptivity has many different paths through which to introduce a disturbance into the boundary layer. These include the interaction of freestream sound or turbulence with leading-edge curvature, discontinuities in surface curvature, or surface inhomogeneities. Moreover, the picture for 3-D flows is expected to be different than that of 2-D flows. Essentially, the incoming freestream disturbance at wavenumber $\alpha_{fs}$ interacts with a non-homogeneity of the body causing its spectrum to broaden to include the response wavenumber $\alpha_{TS}$. Small initial amplitudes of the disturbances tend to excite the linear normal modes of the boundary layer which are of the T-S type (Mack 1984).

It is believed that the vortical parts of the freestream disturbances (turbulence) are the contributors to the 3-D aspects of the breakdown process (Kendall 1984, 1998) while the irrotational parts of the freestream disturbances (sound) contributed to the initial amplitudes of the 2-D T-S waves Kosorygin et al (1995). Thus, freestream sound and turbulence present a different set of problems in the understanding, prediction, and control of boundary transition and, as
such, each require unusual experimental and computational techniques.

**Direct Numerical Simulations** Transition is a spatially evolving process and the spatial direct numerical simulation (DNS) approach is widely applicable since it avoids many of the restrictions that usually have to be imposed in other models and is the closest to emulating experiments. For example, no restrictions with respect to the form or amplitude of the disturbances have to be imposed, because no linearizations or special assumptions concerning the disturbances have to be made. Furthermore, this approach allows the realistic treatment of space-amplifying and -evolving disturbances as observed in laboratory experiments. Reed (1994) comments on the various numerical methods, formulations, initial conditions, disturbance inputs, freestream and far-field conditions, and downstream boundary conditions used in spatial simulations. Reed et al (1998) cover validation issues.

**Leading-Edge Effects** With the spatial computational method, finite curvature can be included in the leading-edge region; this feature was left out of some early unsuccessful receptivity models. Lin et al (1992) demonstrated this by varying the aspect ratio of the elliptic nose on a flat plate from 3 (blunt) to 9 to 40 (very thin) and showing that the vorticity tends to become singular. By stipulating the plate to have finite curvature at the leading edge, the singularity there is removed and a new length scale is introduced.

Experimentally, the most popular model geometry for receptivity has been the flat plate with an elliptic leading edge. Thus it is reasonable that computational models consider the same geometry. However, the curvature at the juncture between the ellipse and the flat plate is discontinuous and provides a source of receptivity (Goldstein 1985; Goldstein & Hultgren 1987). Lin et al (1992) confirmed this computationally and then introduced a new leading-edge geometry based on a modified super-ellipse (MSE) given by

\[
\left[ \frac{(a-x)}{a} \right]^{m(x)} + \left[ \frac{y}{b} \right]^n = 1, \quad 0 < x < a \quad (11a)
\]

\[
m(x) = 2 + \left[ \frac{x}{a} \right]^2 \quad \text{and} \quad n = 2 \quad (11b)
\]

where \( AR = a / b \), \( b \) is the half-thickness of the plate, and \( AR \) is the aspect ratio of the "elliptic" nose. For a usual super-ellipse, both \( m \) and \( n \) are constants. These super-ellipses will have the advantage of continuous curvature (zero) at the juncture with the flat plate as long as \( m > 2 \) at \( x / b = AR \). The MSE, with \( m(x) \) given above, has the further advantage of having a nose radius and geometry (hence a pressure distribution) close to that of an ordinary ellipse.

**Receptivity To Freestream Sound** Receptivity results can be expressed either in terms of (a) a leading-edge receptivity coefficient defined as the ratio of the T-S amplitude in the leading-edge region at \( x = O(U_s/2\pi f) \) to the freestream-sound amplitude:

\[
K_{LE} = \frac{|u'_{TS}|_{LE}}{|u'_{ac}|_f} \quad (12)
\]

or (b) a Branch I receptivity coefficient defined as the T-S amplitude at Branch I normalized with the freestream-sound amplitude.

\[
K_I = \frac{|u'_{TS}|_I}{|u'_{ac}|_{LE}} \quad (7)
\]

where || denotes absolute value or rms.

Haddad & Corke (1998) argue that the appropriate receptivity coefficient is \( K_{LE} \) because it is based strictly on local properties of the leading-edge region, whereas \( K_I \) depends on the pressure gradient history from the leading edge to Branch I. Moreover, because of pressure gradients, \( K_{LE} \) decreases with nose radius and \( K_I \) increases with nose radius which could lead to some confusion.

These arguments are compelling but utilitarian issues sometimes argue against. For example: (1) it is impossible for an experiment to measure \( |u'_{TS}|_{LE} \); (2) most transition correlation schemes begin with Branch I calculations; (3) the pressure gradient history can easily be accounted for by OSE calculations up to a region near the leading edge.

Fuciarelli et al (2000) and Wanderly & Corke (2001) obtain Branch I receptivity coefficients for a 20:1 MSE over a range of frequencies that can be compared with the experiments of Saric & White (1998). The results are shown in Table 1. There is no significant variation with frequency. The agreement between the computations and the experiment is excellent, and we conclude that each validates the other.
TABLE 1: Branch I receptivity coefficients for multiple frequencies as predicted by DNS and compared with experiments.

<table>
<thead>
<tr>
<th>Case</th>
<th>DNS</th>
<th>DNS</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>90</td>
<td>82—86</td>
<td>88—92</td>
</tr>
<tr>
<td>$K_I$</td>
<td>0.046</td>
<td>0.048</td>
<td>0.050 ± 0.005</td>
</tr>
</tbody>
</table>

Wanderly & Corke (2001) extend the computations over the frequency spectrum at the same Reynolds number for the case of different leading edge shapes. This is an extraordinarily useful result. Not only does one have the Branch I initial amplitude for an amplification factor calculation (given a freestream measurement) but also a very good example of how a leading-edge design needs to proceed.

The DNS results are compared with the theoretical predictions of Goldstein (1983), Kerschen et al (1990), and Hammerton & Kerschen (1996) where the leading-edge receptivity coefficient is found to be approximately 0.95. The results of Haddad & Corke (1998) show that the leading-edge receptivity coefficient has a value of approximately 0.47 for a Strouhal number, $S = r_2 \pi f / U_\infty$, of 0.01. Using the same Strouhal number, Fuciarelli et al (2000) march upstream to an $x$-location of 1/2 the wavelength of the associated instability wave [which approximates the LUBLE region defined by $x 2 \pi f / U_\infty = O(1)$] and predict a leading-edge value of approximately 0.75. In an improved calculation, Erturk & Corke (2001) predict $K_{LE} = 0.64$ at $S = 0.01$ and $K_{LE} = 0.76$ at $S = 0$. All of the computations show a strong decrease in $K_{LE}$ with an increase in $S$ in agreement with theory. In trying to do these comparisons, certain difficulties arise and hence the differences between the calculations is purely technical. The major uncertainty that exists in the DNS (and experiments) is the use of $K_{LE}$ and the choice of streamwise location in the leading-edge region at which the amplitude should be sampled for comparison with the asymptotic theory. However crude these comparisons may be, it is clear that the essential ideas of the asymptotic theory have been validated experimentally and computationally.

Fucciarelli et al (2000) use an Ansatz (the details are in the paper) for finite chord and developed the following comparison of $K_{LE}$ at $S = 0.01$.

TABLE 2: Leading-edge receptivity coefficients for various incidence angles as predicted by DNS and compared with the finite-nose-radius theory.

<table>
<thead>
<tr>
<th>$\alpha_\infty$ (degrees)</th>
<th>$K_{LE}$ DNS</th>
<th>$K_{LE}$ Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.75</td>
<td>1.0</td>
</tr>
<tr>
<td>5</td>
<td>1.3</td>
<td>1.8</td>
</tr>
<tr>
<td>10</td>
<td>2.1</td>
<td>2.6</td>
</tr>
<tr>
<td>15</td>
<td>3.2</td>
<td>3.4</td>
</tr>
</tbody>
</table>

The agreement is excellent and clearly demonstrates the importance of including the effects of the finite nose radius in any receptivity study.

Erturk & Corke (2001) carry out the calculations for the semi-infinite body at different Strouhal numbers. They demonstrate the same qualitative behavior as predicted by theory. This result is particularly useful for interpreting experimental results where the sound incidence angle can vary.

**Observation** For each configuration under consideration, the complete integrated picture of geometry and associated pressure gradients (both favorable and adverse) must be included in any meaningful evaluation of receptivity, and it is here that computations by spatial DNS can excel. A variety of different freestream disturbances can be implemented with this technique and the response of the boundary layer quantified and catalogued.

Spatial simulations are still too expensive to use for routine design and at present we cannot provide a completely resolved solution all the way through transition to turbulence even on a flat plate. However, an important and exciting role for the simulation is in the development and calibration of simpler models. The abundance of information provided is invaluable and complements any experimental effort.

Moreover, these results provide the link between the freestream and the initial boundary-layer response and
can provide the upstream conditions for further DNS or Parabolized Stability Equation (PSE) (Herbert 1994) simulations marching through the transition process toward turbulence.

CONCLUDING REMARKS

To conclude, the authors wish to make the following points:

Complementary Efforts

Verification and validation are particularly critical in the study of transition, and advances in CFD will not obviate the need for the wind tunnel and flight. Rather it is clear from recent successes in the research community that advances in prediction methods and in identification of basic mechanisms will come from those groups working hand-in-hand:

- Perform complementary computations and experiments on same geometries
- Because of sensitivity of transition to initial and operating conditions, computations provide validation of experiments and vice versa.

When computations work with experiments, explanation of mechanisms at work is easier to determine and simpler models thus developed. Each provides different level of detail and perspective.

An example of a successful precedent is the one described in this paper where a team at Arizona State University studied basic mechanisms in incompressible swept-wing boundary layers. Here experiments and computations efficiently sorted out the effects of curvature and nonlinearities on breakdown and together elucidated a promising approach to transition delay in these flows through distributed roughness. This promising technique is currently being evaluated for supersonic flight (Saric & Reed 2002, 2003).

As we aspire to understand the effects of freestream disturbances and transition in high-speed, flight-Reynolds-number, and complex-geometry flows, this kind of collaboration becomes even more critical. Detailed measurements are more difficult and costly in these flow. Here, computations can guide the experiments as to what effects are important and what needs to be measured. As a good example, for flat-plate flow, computations have identified new breakdown mechanisms in high-speed flows different from those found in the seminal work of Klebanoff et al. (1962) and Herbert (1988) in low-speed flows, e.g. "oblique-wave breakdown" (Thumm et al., 1990).

The good news is that the CFD formulations validated to date demonstrate that if the environment and operating conditions can be modeled and input correctly, the computations (NPSE and DNS) agree quantitatively with the experiments. What is especially significant and exciting is that the NPSE, which have significantly less resource overhead associated with them compared with DNS, have been shown to accurately model a variety of relevant flows.

Recommendations for Experimental Databases

Validation requires comparison with careful archival experiments, but few such experiments have been performed. Saric (2001) points out to those performing experiments that it is critical for an experimentalist to completely document the flowfield as a companion data set to transition measurements. This includes physical properties, background disturbances, initial amplitudes, and spatial variations. He also suggests that regardless of whether the experimental objectives are transition control, 3-D, secondary instabilities, nonlinear breakdown, or receptivity, the linear problem must be correct. That is, if one can show the comparison with linear theory for the particular flowfield, then the experimental basic state is probably as advertised. A measurement of transition Reynolds number is also advised.

Comments on the $e^N$ Method

Although a great deal of faith is placed in the use of the $e^N$ method for transition prediction in design, it must be used with caution because of several limitations. Care must be used in selecting which correlations to use for any given flow, since stability and transition behavior can strongly depend on the details of the flow. Without a clear understanding of the detailed physics and which instabilities are responsible for transition, incorrect results are obtained. Other complications arise in integrating the linear-stability growth curves. For example, with the crossflow instability, it is unclear whether one should integrate the growth of the mode with the highest local amplification rates (envelope method), or track the growth of a single mode. An even more serious deficiency is the fact that the $e^N$ method entirely ignores the effect of receptivity on the transition process. Higher initial disturbance amplitudes will certainly lead to earlier transition. Radeztsky et al. (1993) indicate a change of almost 5 in the transition N-factor for stationary crossflow as a result of changing the surface-roughness characteristics of a swept airfoil.
Arnal (1994) indicates that the eN method works very well under some circumstances in 2-D boundary layers, but clearly this depends very strongly on the detailed conditions of the experiment. It is not so successful for 3-D boundary layers. Successful transition-prediction schemes must be based on a thorough understanding of all aspects of the transition process.

The strength of linear theories is in their use for design by comparing growth rates and N factors from one configuration to another or doing parametric studies. A configuration with a smaller N factor (using the same form of the theory) is likely to remain laminar longer. If the theory at least qualitatively contains the appropriate relationships, it can be a practical and efficient tool in the evaluation of new airfoil shapes for wings.

Promising areas

Perhaps the most promising area of research is in transient growth. Recently, considerable work in this area has greatly expanded our understanding of different paths by which transition to turbulence can occur and offers an explanation for a number of examples of bypass transition. Of particular interest are explanations of Poiseuille pipe flow, roughness-induced transition, and the blunt body paradox (Reshotko 1999).

ACKNOWLEDGEMENTS

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American Institute of Aeronautics and Astronautics


Figure 2. Streamwise-velocity contours for NLF(2)-0415 45° - sweep, $R_e = 2.4\text{ million}$, $\lambda_z = 12\text{ mm}$, 45% chord. Excellent agreement between NPSE with curvature and experiments.
Figure 3. $N$ -factors for NLF(2)-0415 $45^\circ$ - swept airfoil, $R_c = 2.4 \text{ million}$, $\lambda_z = 12 \text{mm}$. Shown is the excellent agreement between NPSE with curvature and the experiments.

Figure 4. Mode-I velocity fluctuation contours (a) Figure 7 from Malik et al (1999), (b) Figure 20b from Wassermann & Kloker (2002), and (c) Figure 11 from White & Saric (2002).